KLEIN BOTTLE LOGOPHYSICS, THE PRIMEVAL DISTINCTION, SEMIOSIS, PERCEPTION AND THE TOPOLOGY OF CONSCIOUSNESS

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1. INTRODUCTION (an extended abstract)

Upon considering the evanescent character of the primeval sign introduced for invoking a distinction, as done by Spencer-Brown in his Laws of Form, it is the intermittent evanescence of the subject which is brought to the fore, a subject who has dislocated his identity by projecting to the sign, and yet kept apart if not aloof. This reminds us the incomplete character of consciousness, the experience that there is a seeing of seeing or a seeing of the experiencing itself which at certain point of the realizations of the sign is experienced as transcendental but yet requires a closure, as the Klein Bottle's surface selfpenetration. Evanescence is manifested in the thread of thoughts and still on the mutual penetrations of the material-energetic-in-formational fields, an interwoven world of processes –as the HyperKlein Bottles.

Thus we are led to two things: on the one hand the complementarity of the sign and the designer-bearer of it, as two which are one and still two (twoness as oneness and the converse), and secondly, that this requires abandoning the plane as the geometrical locus for the sign to act as a boundary by enacting a Klein Bottle cycle of selfpenetrating return, which we shall point to the signature of consciousness,the Self. However, this evanescence is that of the continuously operating identification which actually takes the sign out of the plane as the subject himself is and thus the intermittence is realized cyclically by selfpenetrating the identity, the Klein Bottle. While keeping restricted this to the plane, the distinction produces a boundary, and the operating intermittence stands for the classical dual logic of Aristotle formalized algebraically by Boole. So there is a naissance of an Outside and Inside relative to the sign and the identification of the subject, a most primitive image-schema which supports much of our organization of our thinking, actually to the point of making its usage transparent to its signification, actually an habit. Be as it may, this becomes the actual generation of perception and cognition, and placing the intermittence as the fusion of them with action, and ultimately the operations of consciousness as a topology of Being, as intuited by Heidegger, at the very foundations of phenomenology. Remarkably, this cyclicity is known to be at the very foundation of theoretical physics (Dolce,2011) as a torsion geometry both of physical space, biological organisms and still to cognition and the non-orientable topologies, Moebius strip and Klein Bottle associated to them, and still associated to selfreference and heteroreference as interwoven principles (Rapoport, 2011,2013). Torsion geometries embody a notion of geometry which differs from the classical notion of space and time as *containers*, say as in General Relativity or Newtonian physics, curvature being the deformed manifestation of the occupying trace of matter, an attribution rather than an ontological condition (Rapoport, 2014b). Rather than physical systems as occupiers of spacetime, physical systems *are* torsioned spacetime geometrical configurations, the primeval vortex for form. So is the case of random Brownian processes, quantum systems, fluid dynamics, condensed matter and particularly liquid crystals –crucial to life, the fields corresponding to the fundamental interactions. Torsion geometries are associated to non-orientability, which embodies its selfreferential topological form

(Rapoport, 2013); it is further crucial to the Topological Chemistry paradigm (Flapan,2000), the chemical bonding and nanostructures (Mihai & Ori,2020), physical or chemical systems as closed knots on tori (Avrin,2011,2012). However, the torsion geometry appears not only to be basic to physics, chemistry and still to biological development and genomics (Rapoport, 2012b, 2016III), but the very basis of a supradual logophysics in which the mind and the physical world operate as a unity, which was developed by the present author.

The Planck constant gives the fundamental scale to the closure of cycles produced by torsion, and further related to spin (Ross,1989). Yet, this is not circumscribed to the domain of quantum physics as its minimal scale, including the topological order of condensed matter (Wen, 2016) whose geometry is not the metric geometry of General Relativity which by default has null torsion (GR), but torsion geometry (Kleinert,2008). It also encompasses the subPlanckian scale of the Quantum Hall effect and its fractional form which is based on the Moebius strip for shape (Meseguer,2018). It further encompasses the level of the vacuum quantum fluctuations conceived as a quantum foam with metric singularities, "black holes", whose event horizon has a Moebius strip for topology ('t Hooft,2018). Neither is circumscribed to physics, since it also involves a quantization of logical momentum (Stern, 2000;Rapoport,2009,2011d), the torsion of cognitive space produced by the non-duality of the True and False operators of Matrix Logic, embodied as the non-orientability of the Klein Bottle, to be discussed later.

In this article, we wish to honour George Spencer Brown, the unique polymath. Thus, it seems fit to focus our contribution to aspects which though related to the underlying conception in the Laws of Form, expands the Klein Bottle logophysics previously mentioned. To wit: to elaborate it with regards to the semiotic nature of the world-as-experience, perception and cognition, the bearing of selfreferentiality and heteroreferentiality in the development of thinking and language, the relation to ecological psychology and ecological information, its bearing in the construal of culture and societary complexity, and still to discuss the relations with phenomenology and the topology of the mindbody. As an example, we shall present a topological-geometrical account on the modelization of perception-cognition by Shepard, an extraordinary transdisciplinarian work understood as a model of the workings of the mindbody in the construal of representations of the world-as-experienced, which has been largely ignored but for the relations with music cognition, as seen from the latter's specificity. We shall contrast this with the torus model of the mindbrain and its hierarchical operational architecture, proposing instead an harmonic-related architecture supported by these non-orientable surfaces, where harmonics is proper to their own shape.

2.THE PRIMEVAL VORTEX AS THE ARCHETYPICAL OUROBOROS AND KIN: HYPER-KLEIN BOTTLES, MOEBIUS STRIP

We have already discussed that the primal distinction establishes a boundary for systems in terms of the Inside/Outside categorization which is a reduction of the Klein Bottle logophysics. (Rapoport,2014b,2016 I). The latter operates as a semiotic agency creating a world, acting and producing signs operating on signs yet one which appears as not devoided of meaning.According to Hoffmeyer:

"[F]rom a semiotic point of view the decisive step in the process that led to the origin of life was the appearance in the world of a new type of asymmetry, 'an asymmetry between insides and outsides'…. The formation of a closed membrane around an autocatalytic closed system of compounds… Such a stable integration of a self-referential digital coded system into otherreferential [i.e. hetero-referential] analogical coded system may perhaps be seen as a definition of life" (Hoffmeyer, 2008). Remarkably, the Klein Bottle in its selfpenetration as a surface embedded in 3D-Euclidean space embodies a joint digitality and analogic continuity, rather than pla-

cing a dichotomy between digital and analogic, it allows the former to code for the latter in its relationality as locations vis-à-vis the selfpenetration. So, the notion that information is the ultimate ground for the constitution of the world and its operations, and that of organisms as well, on the one hand allows for a digital codification, and on the other this is done supported in the analogic continuous character of the Klein Bottle surface: A non-paradoxical constitution of a fusion of discreteness and continuity that emplaces a different ontology and epistemology given by classical dual logic, say as in the 'It out of bit' of Wheeler's reductive physicalism, yet which approached from the latter's persistence on emplacing Inside and Outside as dichotomic, in this latter view is fully paradoxical (Glattfelder,2016). So, if the paradigm for science is to be flagged open as "Information", this turns to be a rather richer one than the mere codification in digital terms, already questioned its applicability to the brain (Buzsáki,2019). It is an ecological information, in the first place, one which is borne from contextuality through embodied experience rather than a mere consideration of context as an Exterior constraint, very much alike Goethe's contextual based theory of colours or still, in JJ Gibson's consideration of perception as fused with action –the ultimate physics concept, now upgraded to a psycho-physics in which a logophysics as a fusion of logic and the physical domain, which emplaces ecological information as its form.

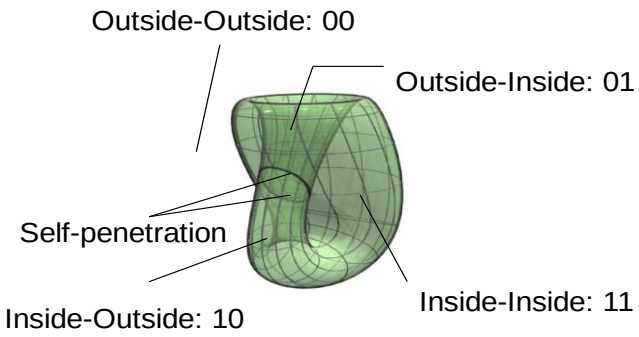

Fig 1. The Klein Bottle and its associated 4-state logic. The generation of genomes start with C as 00, A 01, U or T 10, and G 11. It generates the basic 64 codons of DNA and their representation in terms of the Dirac Algebra of Quantum Mechanics, in the framework of the Nilpotent Universal Computational Rewrite System (NUCRS), a self-referential syntax that generates much of mathematics and the fundamental symmetries of physics in terms of hypernumbers (Rowlands,2008). The Dirac Algebra, its Structures, and Dualism in Physics: On the one hand the *double* 3d space (space coordinates and momentum) of NUCRS, and on the other the double tetrahedron codification introduced in NUCRS, are associated with a pair of KBs, yielding the double helix, which thus appears related to a pair of KBs, being the case that only one is necessary to construct the genome; we showed that the primeval depth time-variable of the KB's selfpenetration (Rosen,2006) materializes as 3d-space (Rapoport,2011b,2016 III); see link to movie in Section 11. Thus, the Dirac Algebra is derivable from two KBs, and the purported dualism of physics as presented in NUCRS, is rendered to be based on the supradual KBL .

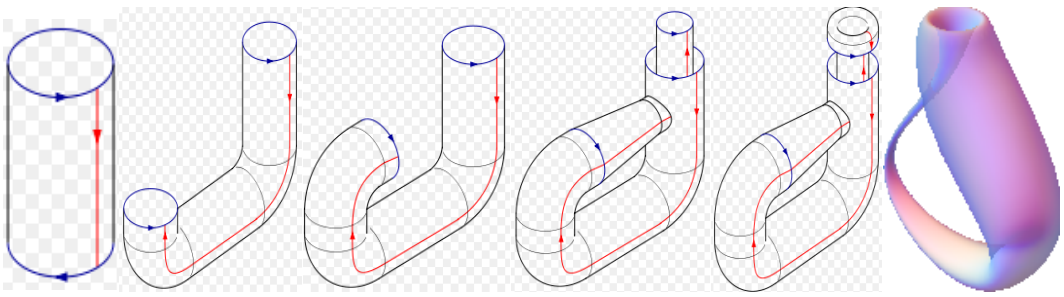

Figure 2 Sequence of topological transformations leading to produce the Klein Bottle - with slight transparency, at the rhs; rendered with Mathematica 8 using the parametrisation provided by Robert Israel; uploaded by Wridgers . Creative Commons SA BY 3.0 File:Klein bottle translucent.png, Wikipedia.

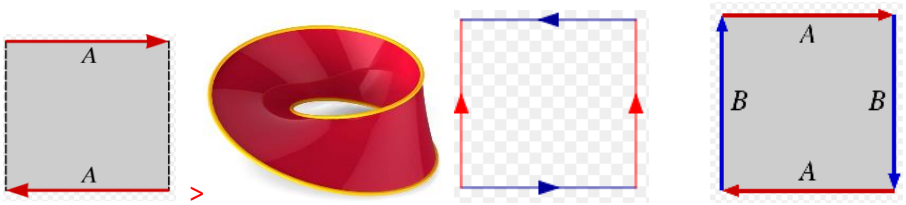

Figure 3 : To turn a rectangle into a Möbius strip (presented at the centre) –in this case a left handed one though right-handed Möbius strips are also the case, join the edges labelled A so that the directions of the arrows match. We note that due to the 180 º twist of the red line in [A], the Möbius strip can be conceived as a dimensionalization producing process: namely, a two-dimensional surface which is contained in a one-dimensional single closed curve (now painted in yellow in [B]); (JoshDip BY-SA 30; MobiusJoshDif.jpg in Wikipedia) The Möbius strip as a surface is contained in three dimensional Euclidean space. [C]: To construct the Klein Bottle glue the red arrows of the square together (left and right sides), resulting in a cylinder. Notice the asymmetry introduced by the cross-identification of two opposite sides coexisting with the symmetry of the other two sides. To glue the ends of the cylinder together so that the arrows on the circles match, you must pass one end through the side of the cylinder. Note that this creates a circle of self-intersection in which the surface selfpenetrates-see fig.2; this is an immersion of the Klein bottle in three dimensions. But in distinction with the Möbius strip, the dimensionalization is such that two opposite lines (depicted in red and blue in [C]) gives rise to a surface which due to the self-penetration, is still embedded in 3d-space but rather than contained in it without self-intersections as for the Möbius strip, it is self-contained, while still being able to act as a container, albeit an imperfect one; it may leak. [D]: Fundamental polygon of the real projective plane; it is non-orientable and non-self-intersecting, in distinction with the self-penetrating Klein Bottle. The Möbius strip can be conceived as a real projective space which one of the pairs of opposite sides identification is frozen.This will be crucial to the forthcoming.

[A] [B]

Figure 4.The cutting of the Klein Bottle and a HyperKlein Bottle. In [A] we see two oppositely twisted Möbius strips produced by cutting the Klein Bottle along the longitudinal section; conversely, zipping them we obtain the Klein Bottle. Thus, in distinction with the Moebius strips which can be either left or right handed, the Klein Bottle is neither, yet possess inherent to it both chiralities–courtesy of Theon (File:Bouteille_Klein_2Mobius.png, Wikipedia)-

[B]HyperKlein Bottle: three Klein bottles set inside each other to produce when cut three pairs of single-twist Mobius strips. CC-SA4.0 https://collection.sciencemuseumgroup.org.uk/objects/co415792/klein-bottle-1995-single-surface-model.Notice that the middle element stands as a non-classical relation with the other two. Furthermore, this does nos stand for a hierarchy but an heterarchy. Indeed, the outer-most KB reenters itself through the innermost KB (the lhs outermost handle) and in doing so mimics the rhs two handles which are intermediators, figuring Firstness and Secondness, thus altogether representing Thirdness to be introduced below. This corresponds to the coalescence of Peirce's triad, Firstness, Secondness and Thirdness, in the constitution of a world of signs, to be discussed in the sequel.Generally, HyperKlein Bottles are embodiment of heterarchies, superseding the hierarchical ordering, yet in some instances incorporating them relative to heterarchies. More and very diverse examples of these self and hetero-penetrating surfaces, with several boundaries rather than a single self-penetrating one, which we generally call as the Hyper Klein Bottle, were produced by Alan Bennett, for the Museum of Science, London, see http://www.scienceandsociety.co.uk/ (search Klein Bottle, see those having multiple reentries).

The Klein Bottle as a metaform for cognition and thought supports in-betweeness, and thus it is the logophysics that is created by this which actually operates as a logic –in the most ample term- of creativity for material, informational and imaginal elements. Not only it surmounts the dual logic and the world created by it –while keeping it as a particular case of itself (Rapoport 2011a,d)- but also reveals an integration of continuity and discreteness, analogics and singular terms, reality, subjectivity and the imaginal domain as integrated, rather than fractured.

3.SELFREFERENTIALITY, LANGUAGE, THE CONSTRUAL OF HIGHER ORDER COGNITION

Chomsky developed a Generative Grammar to support the uniquely creative aspects of human language as the capacity of humans to understand and produce infinitely many novel sentences by applying selfreference in the form of recursion In Chomsky's view, a child's brain automatically begins to process the complex linguistic data to which it is exposed at birth and spontaneously generates a grammar (Chomsky, 1991). This view was highly influenced by the notion of the brain as a computer, which runs on a precharged genetically inherited program.

Actually, selfreference is conceived as the very principle for thought as associated to a supra-dual Klein-Bottle logophysics (Rapoport, 2009,2011as,d,2014b). Selfreference in its operational recursion form supports the recursive generation of language proposed by Chomsky, and much more as we shall see.

The Klein Bottle logophysics is crucial to music and language, both in their propositional, non-propositional and-in-between-them modes (Rapoport 2020).In the non-propositional form, it is remarkable that the Moebius strip appears to be the topology which embodies the phenomenon of pitch cognition, as the Tritone Paradox (Rapoport, 2013,2014,2016). Actually, it was the realization by Shepard of the non-linearity of music perception that triggered his entire Complementarity Principle as the basis for the modelization by the mind (Shepard, 1964, 1981,2017)

In Chomsky's setting language would have appeared based on the selfreferential operation of thought as recursion, rather than thought as an outcome of language. Vygotsky already introduced the notion of an interior language (I-language) of the mind later retaken by Chomsky, and still an exterior outspoken –in the literary sense-language (E-language) very much influenced by socialization and the need to make communication effective, rather than innate and fully set by the existence of grammar. While I-thought is non-linear E-language bears this non-linearity as a sequential linearity subjected to the constraints of grammar (Vygotsky,1986). Corballis: "language does not appear fully formed in different cultures as a product of universal grammar, but comes about gradually as a product of culture and accumulated experience, and

a practical concern to make communication more efficient. That is, it grammaticalizes itself [selfreferentially]" (Corballis, 2011). Whatever, the main difference according to Corballis that sets humans from animals apart is recursivity. It is present in I-language and articulating E-language sequentially under the constraints of grammar, while selfreferentiality in relation of selfhood operates fused with heteroreferentiality: the fact that E-language is borne on culture and experience, places alterity working in tandem with selfreferentially, the Hyper-Klein Bottles.Two main characteristics can be discerned as an outcome of this: Time travel as we can examine the past and anticipate the future, and still a 'theory of mind', the actual inferential process of interpreting the alter's thoughts, behaviours and intentions, which makes of the alter's mind a world capable of being explored and subjected to cognition. A 'theory of mind' is fundamental to the establishment and production of culture as shared knowledge: selfreference and hetero-reference as the basic principle for both the material, cognitive and imaginal world of culture (Rapoport, 2014b,2016 I; Rapoport & Perez,2018; Corballis,2011; Dunbar, 1998). Corballis represents this construal of higher-order cognition as a Russian Matrioshka recurrent hierarchical embedding, however the fact is that the implementation of science affects deeply the other prior elements of the ascending-in-complexity ladder, as notoriously experienced nowadays. Rather than a Matrioshka, as argued for the brain-mind in (Fingelkurts et al,2010,2013,2016) and organisms (Meijer et al,2020), it is a intermmingling of selfreference and hetero-reference, which even may not conserve the order of relationship: a dynamical Hyper-Klein Bottle. It is ecological information what operates in this unending construal of complexity, rather than "It out of bit". The is somewhat scantily realized nowadays; see Conclusions in (Floridi,2011).

However, recursivity is not only about humans. Other species seem to know and operate recursively, as is evident in the humpback whales capability of blowing out and "repairing" Golden Mean spirals for the purpose of creating an enclosure for fishing, as a unified pod-school activity (Rapoport & Perez,2018). As already argued by Corballis recursivity of I-language does not necessarily manifest as E-language, the latter having appeared not in terms of vocalization but of gesturing, of which the humpback whale packs builtup of fishing image-schemas seems to be the case: a conceptual-intentional practice proper of Chomsky's Faculty of Language in the Broad Sense, a means intended to purposeful communication. In Chomsky's theory the fundamental property of I-language is recursion, and in his later Minimalistic Program it is a topological operation that builds up E-language: Merging. Remarkably merging is the topological operation subjacent to Hegel's dialectic logic (Dimitrov, 2017). To resume, recursivity as a formal selfreferential operation is necessary but perhaps not sufficient for language in different species, however for humans Corballis argues that it is both necessary and sufficient . Still, it is crucial to music cognition (Payeron,2011).

3.1 Selfreference, Metaphorization and the Topology of Meaning

However, selfreference does not solely apply to the conception of Chomsky, which is very much attached to language as a syntactic product. In this take syntax and the ensuing grammar produced by the rules of syntax is primal and semantics as meaning construal is somewhat separate if not independent altogether. In cognitive semantics it is conceived that semantics cannot be separated from grammar. Gardenfors referring to Chomsky: "grammar is a formal calculus, which can be described via a system of rules, where the rules are formulated independently of the meaning of the linguistic expressions. Semantics is something that is added, as a secondary independent feature, to the grammatical rule system. Similar claims are made for the pragmatic aspects of language" (Gardenfors,2010). Cognitive semantics upholds that meaning, rather than being about truth conditions in possible worlds, be that real or imaginal,

is related to the conceptualization in a cognitive model. So meaning precedes truth, very much alike as in the Klein Bottle logophysics. Further, since the cognitive models of our brain-mind are related to perception, meaning is not independent of perception, rather than as in the realism which claims that meaning arises as mapping the world(s) through language.

Gardenfors: "…we can translate between the visual form of representation and the linguistic code. A central hypothesis of cognitive semantics is that the way we store perceptions in our memories has the same form as the meanings of words… *Semantic elements are based on spatial or topological objects (not symbols that can be composed according to some system of rules).* In contrast to the Mentalese of Fodor and others, the mental structures applied in cognitive semantics are the meanings of the linguistic expressions; there is no further step of translating conceptual structure to something outside the mind. Furthermore, instead of being a symbolic system having a syntactic structure like "Mentalese", the conceptual schemes that are used to represent meanings are often based on geometric or spatial constructions" (p.22, ibid).

While in the usual take metaphors and metonyms are adornmental figurative construals, they are crucial to enrich cognitive models. The latter are primarily image-schematic (not propositional) (Johnson,2007;Lakoff & Johnson, 2003). Finally, and in a marked distinction with Chomskian linguistics, semantics is prior to syntax since it exists as perceptual representations, which exist already prior to the full development of language as the branch of linguistics and logic concerned with meaning, given that semantics operates on geometrical and topological objects? To elicit a response to this query, we turn to semiotics, the study of sign phenomena, particularly to natural human language– the semiotic phenomenon par excellence. But the query pushes us to consider the semiotic process itself, rather than considering signs per se. Following the topological consideration of Being somewhat inaugurated by Heidegger, who placed situationess as the key element of his phenomenology who let itself in principle to propose it to be formulated as topology (Malpas,2007) we turn to the topological consideration of what makes the signification process possible at all, as the primitive form of meaning construal. This leads us to the doctrine of Radical Recursion due to Rosen, the philosopher who inaugurated the Klein Bottle topological philosophy (Rosen,2004,2006,2008) further persued by Rapoport (2014b).

As Heidegger considered that his phenomenology was founded on topology and location, which he stayed short of identifying them, as well as elaborating how and why it was so (Malpas, 2007), the role of topology with regards to sign systems, semiotics, is still more notorious in biology (Barbieri, 2008;Emmeche & Kull,2010; Hoffmeyer, 2011). It is in biological systems where signs and codes play a crucial role, and quite a magic at that would not be that topology was at the very roots of their constitution and operation, as already identified in the Klein Bottle topology of the Mendeleev table of elements (Boeyens,2010) and in genomes (Rapoport, 2016 III) and still in their unified being elicited by Perez (Perez,2015). This conception goes back to a proposal by Pattee, who came to consider folding (without identifying it as of a topological nature and protocols) at the very origin of life. As stated in (Rapoport & Perez, 2018): "For…Pattee for whom life is matter controlled by symbols which themselves are in charge of the physical instantiation, it appears that at the most elementary level, this instantiation is the byproduct of folding, which in the supradual logophysics is of *topological* nature. Thus, the *function* of the measurer or function of the control system, in Pattee's conception, is identified with the folding operation itself. In our conception, we can put it in other words, to the topological process of identification". Still,

"Remarkably, Pattee makes of selfreference the obstacle for control systems to operate as measuring devices to the effect of control. Indeed, he considers this to introduce instead his

“epistemic cut” between the measured and the measurer – a replay of Russell’s Paradox argued from dual logic in his Principia Mathematica to banquish self-reference from the foundations of Mathematics…, the hegemonics of dualism, again. The problem with this conception, is that it thus falls to an infinite regress of systems measuring systems, so that instead of the open ended causality of measured and measurer of dual logophysics, alike an unending sequence of Maxwell`s Devil, the necessary “closure” has the self-referential self-penetrating topology of the Klein Bottle …This is just the same that the slippage of the inter-pretant vis-à-vis the sign that postmodernists’ take on semiosis produced and Rosen healed by proposing a radical recursion, so that the interpretant and the sign become united as a Klein Bottle rather than falling into an unending chain of interpretants making sense” (Rapoport & Perez,2018).This “magic” of selfreferential wholeness of semiotic systems which are further integrated into a single one, is what allows an organism to selfregulate and operate to reach for and conserve an homeostasis which manifests at a most primal level as its emotics, and at higher-cognitive levels as fully developed consciousness (Damasio, 2018, 2018,2018). “Thus, an organism operates as a network of Maxwell demons, each measuring what is in their own purview, and making order out of disorder, according to its own criteria of what is acceptable to itself and what is to be rejected” (Rapoport & Perez,2018). The crucial role of constraints in the operations of the mind-brain as a creator of experience and language has lately been highlighted in (Buzsáski,2019).

In the classical elaboration of semiotics as introduced by Saussure, he explained that a sign was not only a sound-image but also a concept. Thus he divided the sign into two components: the signifier (or "sound-image") and the signified (or "concept"). For Saussure, the signified and signifier were purely psychological; they were form rather than substance. Nowadays the signifier is interpreted as the material form (something which can be seen, heard, touched, smelled or tasted) and the signified as the mental concept (Chandler,2017). Rosen upon examining the classical understanding of signification, notes that the relationship between the signifier and what is being signified is such that its stability is “ maintained by preserving the anonymity of the former” as a kind of active erasure of the material source but for its effect on elicitation of the signified meaning. Furthermore, in the usual conception of semiotics, it is the meanings that are signified what garners our attention, rather than on the act of signification itself. However, semiotics as a discipline changes this by considering the process of signification, rather than the signs or the meanings that arise in their elicitation. However, the process of semiosis, does not operate in terms of Aristotelian classical logic, in that already the most basic Principle of Identity fails in this regard: signs flow metamorphically rather than having a fixed distinguishible entitity.

Merrell: “Semiosis is the process of signs becoming signs. It is the becoming of beingness and the beingness of becoming of both minds and worlds, without the possibility of beingness becoming fixed, accordingly with the Principles of Identity…” (Merrell, 2010). Remarkably, this potential plurality of beingness is the case of molecules, as considered in the Topological Paradigm of Chemistry, as discussed in (Rapoport, 2016) and particularly in biochemistry and genomics (Rapoport 2012, 2016III).

So we turn to the semiotics as the study of sign phenomena, particularly to natural human language– the semiotic phenomenon par excellence. But the query pushes us to consider the semiotic process itself, rather than considering signs per se. Following the topological consideration of Being somewhat timidly inaugurated by Heidegger (Malpas,1999,2007) we turn to the topological consideration of what makes the signification process possible at all.

Rosen indicates to selfreference operating as the principle that underlies semiosis and to the nonlinear cyclicity of this process, as identified separately by Merrell. Rosen: “Here [semiotic

process] the sign becomes recursive; instead of focusing exclusively on signified meanings, it comes to focus on itself. The signifier, which had played a predominantly tacit role in classical semiosis, is now itself explicitly signified" (Rosen,2004). We have claimed that the torsion vortical geometry associated to self and heteroreference has a protosemiotic nature in instauring ciclicity as the fundamental motion of nature and cognition (Rapoport,2011, 2013,2014,2016).

We recall Spencer-Brown's observation that the primeval distinction posed by the subject becomes identified with it, and interchangeably at that, a participant not only in enacting the distinction but indistinct to it, and yet seemingly detached to it.

In other words, the signifier selfrefers in the process, rather than solely standing as the source-reference for others. However, though the procesual conception changed the focus of our understanding of semiosis, by considering it as a process, those who maintaned themselves in the structural interpretation, sought to ascribe themselves to preserve that invariance which the principle of identity supports, to preserve the invariance of the link between the given signifier and what it signifies. The problem is that, once classical signification is surpassed by signifying the signifier, i.e. when the process is purportedly complete, the door is opened to an infinite regress, rather uncompleted. Rosen:"For now, it seems that no signifier is exempted from mutation into that which is signified"; thus the 'sacrosanct' dual logic principle of identity ceases to apply, a metamorphoses is in the making, and it is recurrently so. Still, "a new signifier is presumably needed to signify what *had* been the signifier, but this new signifier is subject to signification by a still newer signifier, and so on *ad infinitum*. And each time the tacit operation of the signifier is undermined by being explicitly signified, the functioning of what had been signified by that signifier is also affected. Ultimately then, we have in this "hall of mirrors" neither signifier nor signified in any stable, abidingly meaningful form" (Rosen, 2004).

A similar regress is produced in cognition, when an homunculus is the assumed case for the construal of cognition, which raises the existence of a chain of homunculi,one observing the other, and thus as a chain of hierarchical Russian dolls ascribing to a recurring Inside-Outside dual logic and thus cognition is unachievable, the hierarchy does not close, cognition is a chain of unending unassessable representations (Neumann, 2008;Rapoport & Perez,2018). Or still, the problem of overall selfregulation of an organism through measuring control. Rather than this regress, nor a trivial identity between signifier and signified to avoid it, what Radical Recursion proposes is a difference which supports an identity: supraduality, at that.

Rosen: "in Radical Recursion, though the self that is signified is not simply the same self that does the signifying [again, the principle of identity cannot be sustained along the process]; through the very act of reflecting upon the self turns it into what is other; this other flows right back into the source from which it arises, rather than appearing *merely* as an other cast before a new self [the non-linear closed loop of semiosis, early argued for in (Merrell, 1996)]. The semiotic act I am intimating thus would give us neither self nor other, in the categorically opposed sense of these terms. We would realize instead their paradoxical interpenetration. I suggest that this dialectic is what we require to supersede the supremacy of linear signification in a meaningful way [as before, a linear chain of causality is no longer]. Signifier and signified would be more than reciprocally interdependent in such a self-signification. They would be identical, utterly one. Yet they also would be two. By virtue of the latter aspect, meaningful signification would continue; by virtue of the former, recursion would go all the way down; it would be realized concretely in the heart of the actual occasion" (Rosen, 2004).

Thus, rather than the indefinite repetition of recursion that fails to achieve an actual signification, the latter is the case continuously in the present time by selfreturning closure: nonlinear recursion. This twoness as oneness Rosen identifies as the Klein Bottle rather than the Moebius strip, due to its selfrealization by a dialectics of continuity (the signification process)

and discontinuity (the self), the latter being intimated by the self-penetration, all in all, a sign of itself, as pragmatist semiologist Paul Ryan puts it after Peirce. Also Merrell reached for a similar ontoepistemology, independently developed in (Rapoport, 2014b).

So while in Chomsky's linguistics is selfreferential as syntactical recursion, the very possibility of eliciting meaning of this somewhat mechanical yet non-linear articulation, lies in that the underlying dynamics of this non-linear articulation is embodied as the joint continuity and discontinuity of the Klein Bottle, as the underlying metaform of the semiotic process that supports the construal of meaning.

4.THE NONORIENTABILITY OF SEMIOSIS AND THE PEIRCIAN CATEGORIES.

Charles Peirce developed the foundations for first and second-order propositional logics, and semantics. He did so in terms of simple graphs in which selfreference appears as the iteration and erasure by deiteration of the unique sign of this system. This sign stands for contention/ boundary, and this emergence of inferences bundled into a cognition unit signed by this contention is very much like the articulation of discourse in language, as recursively structured according to Chomsky, and alike thought. Also this recursive articulation leads to the higher forms of thought, and ultimately, the development of arts, science, culture at large, to be discussed below (Corballis,2011). Though Peirce's theory remained as basic contributions to the foundations of propositional logic, their simplicity and extraordinary efficacy were sidelined by the mainstream adoption of Bertrand Russell's symbolic developments, which ultimately lead to his rejection of selfreference and the ensuing collapse of the formalistic Hilbert-Frege-Russell program for a semantic-free syntactic mathematics as per Goedel's theorems (Zalamea,2003).

Peirce's graph logic, though propositional, is a precedent to Spencer-Brown's non-propositionnal calculus of distinctions with a single sign of distinction/boundary, the distinction operating selfreferentially as a container/enclosure. The Klein Bottle is the reentrant self-penetration of a contention/boundary on itself, which requires leaving the plane where the graphs are drawn, and in doing so, becomes the metaform and standing wave in terms of which perception is enacted in the first place, as a pattern formation and recognition process (Rapoport, 2013, 2016, Rapoport & Perez, 2018). As for the role of selfreference in music, and particularly through the Fibonacci series, we refer to Payeron (Payeron,2011), and more importantly, to the Tritone Paradox, below. Remarkably Peirce's topological graphs leads to a notion of a conti-nuum of non-numerical character as a genericity in terms of a relational logic, which recalls music cognition (Zalamea,2003).

Remarkably, it is in the work of a lifetime by Floyd Merrell, a semiotician-philosopher with a unique polymath background, that semiosis is identified with a logic that goes beyond the classical Aristotelian logic, and ultimately with the Klein Bottle. Merrell departs from Peirce's triadic: Firstness, Secondness and Thirdness: " Firstness is what it is, without any relationship whatsoever with any other. It is self-contained, self-reflexive, and self-sufficient; it is a feeling, before there is conscious awareness of what the feeling is a feeling of some other"(Merrell, 1996). Otherness missing, this selfreferentially further free of relation with other is the emptiness, the Void of ancient lores, the empty sign where no distinction has been actually drawn, and which in introducing it defines two possible modes of Secondness.

Prior to discussing this, we must remark that the semiotic processes teem with uncertainty, incompleteness, fuzziness, paradox, all in all qualities that clearly indicate they do not operate in terms of Aristolelian-Boolean logic (Merrell,1996,2008,2010) but a supradual logophysics. Particularly with respect to biological processes which have come to be studied as a novel discipline of biosemiotics. This is also the case of the body-mind, and thus of the neuroscien-

ces, cognition, medical arts, etc. Furthermore, it is so in all accounts which root material processes on information, as has become to be identified as the foundational processes operating even in the constitution of space and time.Most of these studies, if not all but for very few exceptions are developed as implicitly rooted in Aristotelian-Boolean logic, already identified as a hegemonic habit of the mind which is self and or socially imposed (Johnson,2007). Thus, the Peircian categories are open to interpenetration, due mainly to indetermination or vagueness. Of the three Piercian categories, "prototypes" we can note with Merrell, and quite literary at that: "The categories have no clear and distinct boundaries, nor they are static. They slide over, under, and merge into one another, which is to say that their boundaries are slippery and vague" (Merrell, p.9, 2007).

Indeed, if Secondness does not inherit the selfreferentiality of Firstness, then Secondness as the actualization of all possibilities, becomes a particular OAH (object-action- happening, as introduced by Merrell, a being, and this Secondness operates with the logic of particulars: Aristotelian-Boolean two-state classical logic. This is the hegemonic logic of the sciences, and also most of philosophy, politics and management, daily life, etc. However, would Secondness inherit the selfreferentiality of Firstness, then we identify the logic of Secondness as that of the Klein Bottle. As a surface becomes the supradual metaform of Secondness, together with the Moebius strip, the former selfpenetrating embedded in 3d-Euclidean space and the latter embedded in 2d-Euclidean space and nonselfpenetrating. So as a metaprinciple, the actualization of the metapossibility of inheritance of the defining properties of the categories yields completely different ontoepistemologies: the supraduality of the Klein Bottle or the Moebius strip, as a possibility. Or, for another actualization with no inheritance as an additional particularity the dual logic, the particularities interpreted as definitive characterizations and as such indisposed to metamorphosis. In terms of introducing Otherness through a distinction, and a distinction that generates distinctions at that, Secondness generates what essentially is Boolean logic, as shown in the seminal Calculus of Forms due to Spencer Brown. Its metaform defined by the distinction as a boundary is the sphere of any dimension, or particularly the circle drawn on a plane. However, there is an ultimate mathematical transformation that leaves this dual-container being invariant: The eversion of a 2-sphere in a 3D Euclidean space, a selftransformation of a 2- sphere in itself by smoothly –i.e. without any discontinuities produced in the process- of turning Inside-Out and consequently Outside-In, as an abstract metamorphoses which inverts yet keeps the order of dual-logic. This is the Smale Paradox which also is the case of the 2D-torus (Phillips,1966; Max,1997). This quite intricate transformation requires the Moebius strip and still the antipodal projective structure of the 3-sphere (Aitchison,2010), the latter being considered a model of consciousness (Willford et al, 2012;Rudrauf et al,2017), and identified in music and its cognition. However, in Nature, this is indeed the case of a sea creature, immortal Hydra, or the implosion-explosion of a supernova collapsing star from which most matter on the cosmos appears to originate (Rapoport, 2016II),or still the transformation of the fecunded mammal ovum as the process of diferentiation and development of the ensuing organism. To resume, even when we impose on Secondness the particularity of not inheriting the supraduality of Firstness, it appears that the ensuing dual-logic particularisation of Secondness has a potential not elicited in this transition of Firstness to Secondness. Namely the reversibility of containment through supradual transformations which realize the eversion, proving once more, in this particular setting, that dual-logic is supported by supraduality as related to non-orientability. In other terms,nonorientability has a metaphysical primeval character.

The other more general metapossibility of which the dual case is a subcase is through the inheritance of selfreferentiality is through the most archetypical action, that of the vortex

spinning motion and ultimately the vortex of nonorientability, as the Moebius strip and Klein Bottle. Whereas in the dual subcase is given by an action subjected to an Interior or an Exterior, with the pending need to relate one to the other: the dual logophysics of the sciences, and the ensuing quest to resolve it, unsuccesfully, though.

The nature of Firstness corresponds to the pregnancy of feeling, the possibility of a might be, in expectation of action to actualize it. This action comes about through indexicality, the pointing to as a primitive action of introducing that standing-for-something though fully undeterminate rather than the primal sensate. Thus comes about Secondness, by imposition of a distinction: the sign of Spencer Brown rather than the primal sensate, as Secondness. This is the sign of the selfreferential torsion vortex, spinning as the prototypical action, the sign of self, yet as a sign of itself. It entails the transition of the possibility of being, to an actual being, an OAH, yet in the continuous process of becoming something else. It is the actual river of Heraclitus in the process of becoming. Merrell:" Secondness -in the manner of indexicality- is what it is, insofar as it enters into interrelationship with some other, the other of the feeling.

Secondness entails interaction with the other in the sense of something here and something else there, the first something possibly acting as a sign and the second something acting as the object of the sign". Secondness introduces as actual possibility the Other as diversity yet relative to the undistinguished Void. This actual selfcontained selfreferred action introduced by a distinction, otherness, the Klein Bottle, as the very sign of what is, Being.

Merrell, quoting from Peirce: "Thirdness—basically of the nature of symbolicity—is what it is, in the respect that it brings Firstness and Secondness together by mediating them, and at the same time it brings itself into interaction with them in the same way they are brought into interaction with each other (CP 2.227-390)" (see Peirce,1931-1966) . Firstness is to be considered following the demise of the mental habits posed by the tenets of Aristotelian classical logic: the Principle of Identity, the Principle of Non-Contradiction and the Principle of Excluded Third, assumed for Firstness.

Already Gotthard Gunther in the 1960s argued that the abandonment of Tertium non Datur implied prompting a supradual ontoepistemology, having for ontological loci four elements: Being, Image-as-an-Object, Imagination as a process, and Time –as an operator, which Rapoport assigned to the Klein Bottle ontopoiesis and its logophysics (Rapoport, 2014,2016) .

However, a postreflection considering Peirce's Firstness and biological psychology's (Damasio, 2018;Panksepp,2004) surfacing of feeling, drives us to suggest feeling instead of image as an object, the body-mind core of the "sentient self" (Craig,2010) Indeed: "The range of all possible possibilities contains both one term and its perception-cognition *and* its contradictory. Within the flow of semiosis they become a matter of *both-and* …of all possible possibilities one or more of which can emerge Firstness". Whereas Secondness, including the world of particulars is anchored on binary either-or logic, Thirdness is represented by a HyperKlein Bottle that mediates between them (the in-between) but further reentering its own distinction as a mediator and still mimicking the interpenetration of the other two distinctions of Self and Other. Thus, Thirdness is represented by a contorted HyperKlein Bottle introduced by two distinctions. The first one that transits from the domain of possibility of posibles to an actual one, such that mediates between them (the in-between) of Void and Klein Bottle, but further reentering its own distinction as a mediator –and particularized, at that, as Secondness, but still mimicking the interpenetration of the other two distinctions of Self and Other: The figure no. 4 above.

We introduced, following (Merrell,2010) objects-actions-happenings (OAHs), and still the concept of Contradictory-Complementary-Coalescence (C-C-C) well visible in the previous figure as well as that of Interaction-Interdependency-Interrelationess (i-i-i). Finally the idea

that breaks with the Principle of Identity, that everything is in the process of becoming something other than what was becoming (BSO), also present in the figure. A crucial C-C-C is mind-body (reductionists of computational neuroscience vintage would perhaps adventure to offer brainbody) which coalesces the doings of the mind and those of the body, and yet transcends them in being neither one nor the other – as in Nagarjuna's Middle Way (Westerhoof,2009). Thus the mindbody is to the dual ontoepistemology a novelty resultant of the supraduality which rather than posing in-betweenness to the breakage introduced by duality , different to both mind and body, it poses instead a supradual ontoepistemology for which the in-betweenness is a subsidiary instance and manifestation of it, both being worthwhile to understand and further attempt to perceive its manifestations in others and ourselves, the world at large, as well as other dually-produced- breakages such as mind/world, language/world. In noting that the ultimate human nature is a sign, coextensive and coexistent with other signs, as a socio-propioceptive-somatic-kinaesthesic i-i-i,supporting a mutual complementarity and co-participation with other signs including ourselves. Preceeding perception and cognition of OAHs lies feelings, emotions, intuitions and inordinately vague sensations, the matter of Firstness. Remarkably it is this matter which is the core basic element to what in the further interplay with Secondness and Thirdness and their mutual interpenetration, provides a source and support for the appearance of selfreflective consciousness. And rather than being exclusive to humans lies as the continuity of all forms of life, from the unicellular to pluricellular complexity, acting as societies or "simple" associations, with a commonality of "consciousness. This order stemming from Firstness constitutes the "Strange Order of Things", as Damasio called it in his homonimous work (Damasio,2018).

Amazingly, for the dual ontoepistemology which somewhat disociates rationality from emotionality and makes of the former a human case, starting from Firstness this "strangeness" (or better, the estrangement that in a world of dualistic habituation is provoked) finds a commonality of "consciousness" in all of them; see Damasio's extraordinary work, and that of Panksepp's school of biological psychology), are cases in point (Panksepp,2004). Another i-i-i is that of body-mind and worlds and the resultant C-C-C which may be construed as a panpsychism associated to its in-betweeness, or still the body-mind, languages and worlds i-i-i.

Merrell observed that " Thirdness has a short timerate and at times in which tertiary underdetermination realizes an opening of Excluded Middle" which is tantamount to the mediating Inside-Outside and Outside-Inside states of the Klein bottle as the surgence of novelty vis-a-vis the binary habits (Merrell,2008).This brings us to surprise in the face of paradox given the habituation to dual logic, and as we shall see the body-mind operates to reduce surprise through a free energy principle associated to neural networks functioning as Bayesian inference systems (Friston,2010,2012). This is crucial to the body-mind emotics related to a principle of homeostasis (Panksepp,2004; Damasio, 2018).

5. THE ECOLOGICAL KLEIN BOTTLE LOGOPHYSICS AND GIBSON'S ECOLOGICAL PYSCHE

The ecological approach to perception and action pioneered by James J. Gibson (Gibson, 1960, 1979), known as ecological psychology has for one of its fundamental principles the continuity of action and perception. Rather than embracing the several psychological dichotomies of perception/action, organism/environment, subjective/objective, and mind/body which are at the basis of some theoretical assumptions in the field of psychology, such as the poverty of stimulus and the passivity, it attempted to offer a theory which surmounts this duality. Of these dichotomies, the organism/environment is the one that is central to ecological psychology, and in incorporating the diverse cultural and social factors as shared by the subject and the environment, is more comprehensive that the body/environment or body/mind dicho-

tomies One fundamental principle of the ecological approach is that of the organism-environment mutuality, or in Shepard's geometrical-topological terms, the organism-environment complementarity. According to this principle, organisms and environments are not separate or dual-logically distinct entities—the organism-environment system (not just the organism) is the proper, irreducible unit of analysis for understanding functional behaviour (Gibson; Lobo et al, 2018; Turvey,1975). Remarkably, Lobo et al have posited that rather than the organism/environment duality the case is of a Moebius strip unity. A core claim to ecological psychology is that perception arises in terms of perceptual systems, such as the visual system, and still the organism-environment system, specificity, affordances which as we shall see are associated to supradual logophysics. Gibson argued that affordances defined as the possibilities for action that the environment allows for an organism, are the main objects of perception in his theory. Gibson in following the pragmatist view due to James that experience can be described either physically or psychologically, described affordances as "both physical and psychical, yet neither" (Gibson, 1979/2015, p. 121). This is, quite literally, the formula of the coalescence (the "and") of two domains, the material and mind domains ,yet which does not exhaust (the "neither") the domain of ontological existence, an entangled state in the Klein bottle logic, which does not comply with the Excluded Middle, and also, quite literally, corresponds to Nagarjuna's supradual doctrine. This description of affordances cannot be understood without appealing to the influence of radical empiricism, the doctrine that our knowledge originates in experience, and monism, as a manifestation of the supradual ontology of the Klein Bottle logic.

Another concept of ecological psychology is that of ecological information which differs from the notion of information that arose in engineering of communication through digital systems, and associated to dual logic and its logophysics. In this setting, there is a constitutive contextuality of the organism vis-à-vis the environment, the latter providing constraints as context-conditioned variability, that complement the laws of matter and information. These environmental constraints are formative of the body-mind and are interiorized through biological evolution to become innate conditions in terms of which perception and action are framed on. Rather than vision –for one sensorial mode- following the elaboration by the brain in terms of the light quanta impinging the retina, vision as construed in terms of sensation that sets the construal of visual perception, Gibson pointed to vision being construed in terms of information, which arises contextuality as a light *pattern* emanated by a source interacting with its physical environment that leads to the percept being formed directly, rather than by higher-order elaboration upstream the brain. In this Gibson recalls Goethe's critique of Newton's theory of the formation of colours and the importance of boundaries, the contextualization in the appearance of colours in the first place, which Newton had ignored completely (Heisenberg- see Lehrs,1951). Goethe characterized colour as arising from the dynamic interplay of darkness and light, the former defined not in dualistic terms as its absence but its polarity, a paradoxical antipode in a supradual sense, nor light nor the state defined by its absence. Goethe realized that the sensations of colour conceived originally as reaching our brain are also shaped by our perception — by the mechanics of human vision and by the way our brains process information, an eco-logical psycho-physics indeed. Most remarkably, particularly in relation to the so-called hard problem of consciousness studies on the allegedly origin of qualia, Goethe's experiments led him to consider that the sensation of complementary colours does not originate physically from the actions on our eyes but perceptually from the actions of our visual system.

Thus, in the view of the present logophysics, the origin of qualia , if any at all, appears as related to the unity of action and perception, i.e. to the non-orientable logophysics which

supports it.Thus, the "hard problema" of consciousness as to the nature of qualia, finds its setting in supraduality, be that purportedly rooted to physics or innate to a panpsychism.

So Goethe's theory can be conceived as the first known ecological psycho-physics, by which the affordances give the structural and processual perceivable conditions in terms of which the percept is elicited-created and is complemented by action as a Moebius strip unity (Turvey, 2004). It led to the the theory of antipodal colour opponent processes by Ewald Hering. It states that the human visual system interprets information about colour by processing signals from cone cells and rod cells in an antagonistic manner, and based on three opposite pairs: red versus green, blue versus yellow and black versus white, the former pairs supporting the creation of any colour.It is analogical rather than discrete. Rather than assuming that the eye detects each colour individually, it assumes there is some overlap in the wavelengths of light to which the three types of cones (L for long-wave, M for medium-wave, and S for short-wave light) respond, so it is more efficient for the visual system to record *differences* between the responses of cones, rather than each type of cone's individual response. Rather than the dualistic logophysics of Newton who departs from light as a seen object by the retina, Goethe departed from light as experienced systemically.

As such, it reflects the later cybernetic principle due to Gregory Bateson that reality arises from differences producing differences, rather than non-relational discrete information, ecological information indeed, analogical at that. "The renouncing of life and immediacy, which was the premise for the progress of natural science since Newton, formed the real basis for the bitter struggle which Goethe waged against the physical optics of Newton. It would be superficial to dismiss this struggle as unimportant: there is much significance in one of the most outstanding men directing all his efforts to fighting against the development of Newtonian optics." (Werner Heisenberg, during a speech celebrating Goethe's birthday- see Lehrs,1951).

No wonder Goethe was misunderstood and attacked: he was talking about colours as they surge, physics as experienced by the body-mind, psycho-physics, rather than Newton's experience of physics and its theory. Even today, when consciousness has become a respectable scientific endeavour most publications ignore psycho-physics altogether. It is even claimed a "theory of everything" (Meijer et al,2020)- criticized in (Rapoport,2020)- based on non-ecological information and ultimately assuming a dual logophysics.

6. HETERARCHIES, THE KLEIN BOTTLE ARCHITECTURE OF PSYCHO-PHYSICS, AND A CRITIQUE OF THE TORIODAL TOPODYNAMICS AND THE NESTED HIERARCHICAL MODELS OF THE BRAIN-MIND

Our cognition appears to develop in terms of locational relationship as image-schemas (Johnson, 2007;Lakoff & Johnson,2003) which operate as suprastructures which very much determine what and how are we able to think of, actually determinants of ontoepistemologies. The most predominant is the CONTAIN image-schema to the effect of producing a dual logophysics. Another one that constraints our articulations is related to the Top-Down and its converse, Upper-Down: We are introducing hierarchy.Verticality is no abstraction, per se.

With respect to the mind-brain, this can be framed in different ways, one of which is the anatomical evolutionary hierarchy as equated with the functional hierarchy, as in the Mc Lean triune model of the brain under evolution (McLean,1990). Another form is given by considering both in terms of functional processing and organization proper, for both sides of the dual divide, brain and mind, an issue which shall be further discussed in the sequel.

With respect to the former, it is interesting to reflect on the remora of 19th century positivism with the upholding of precortical brain capabilities as supreme forms of the mind operations. Music involves highly abstract structures and thus, for the mammal brain, related to the higher

prefrontal cortex. Yet it is strongly associated to the most primitive limbic layer of the brain related to emotics (Panksepp,2004). Birds together with mammals have the biggest brain in relation with their body, have amazing cognitive capabilities. Ravens can recognize themselves in mirror and still produce a "theory of the mind", anticipating behaviour of others, establish causality and draw conclusions. Parrots and corvids show that their cognitive skills are on par with primates, mimicking of complex tunes, for one. While in distinction with mammals with highly structure frontal cortices, birds with a rather primitive prefrontal cortex produce very intricate musical pieces and mating theatralizations. Yet "the avian forebrain displays no lamination that corresponds to the mammalian neocortex, hence lamination does not seem to be a requirement for higher cognitive functions. Because all other aspects of the neural architecture of the mammalian and the avian prefrontal areas are extremely comparable, the freedom to create different neural architectures that generate prefrontal functions seems to be very limited." (Güntürkün,2005). In other words, the nested structure of the brain-mind is not a prerequisite for higher order cognition, nor for the sublimation of emotics as a higher order cognition. 'Music is the language of emotions', thus its universality (Korsakova, 2019). Music with its merging with emotics produces a semiotic process, signs and emotions involved in metamorphosis and complementation, the sense of selfhood lived in seemingly fullness. With music memories flush into being, as if time lost now recovered. If anything, the form of music cognition is non-linear, and an openess to surprise elicits its potential incompleteness and capability for renewal. No nestedness at that neither.

In distinction, higher order symbolic elaboration as in mathematics may have for outcome a cognition which is wholly untraceable to emotics, and structured as dualism. Upper brain-mind produce does not equate with higher order cognitive integration, neither. No nestedness on this.

However,rather than hierarchies (Fingelkurts et al, 2010,2013), the brain-mind elicits a interpenetration characteristic of heterarchical order interwoven with hierarchy.This interpenetration is natural in its complexity vis-à-vis the notion of enfolded implicate order due to Bohm which though ultimately topological, evokes a complexity (folding,etymologically) which does not consider this interpenetration of Self and Other, which is characteristic of nature and selfhood, and particularly language. We shall discuss this later on introducing the ecological theory of psychology due to Gibson.

Rather than the simple enfoldment of Bohm, the radical form of this operation is the self-penetration of the Klein Bottle and still, the co-operation of selfpenetration with hetero-penetration, rather than the iterated dualism embodied by nested dualism in the form of Russian Matrioshkas dolls (Rapoport 2014b), which we shall reveal in the sequel. The underlying universal case is the mind-body and the environment. The wholeness of the Klein Bottle holds both the implicate order- its selfpenetration, from which with its embedding in four dimensions we can see its generative operation of the whole as explicate, already embodied in its three dimensional embedding, yet which disappears from sight, to cyclically regenerate: an evanescent cycle; see below. No metaphorization through Quantum Mechanics is required nor natural - in acknowledgement of the crucial role of metaphors in cognition (Johnson, Lakoff & Johnson), yet not to be overstretched in replacement for ontoepistemological principles.

Indeed, the Matrioshka's hierarchical nestedness has been claimed to be the case of an operational architecture of the brain-mind (Fingelkurts et al,2010,2013) and related to Bohm's enfoldment 'implicate' and manifest 'explicate' orders. It is further related to Oriental metaphysics, basically to selfreference as the generating principle, for the latter's contribution (Fingelkurts et al, 2020). However, the Klein Bottle and Moebius strips surfaces are metaforms of selfreference, -unmentioned by these authors. They are crucial in conceiving the mind as a

dynamical system undergoing metastable transformations from which order arises, precisely as in the operational architecture previously mentioned, to be discussed further.

Thus, semiosis which is rooted in the supradual Klein Bottle logophysics is not supported by a torus' orientable topology, which embodies a banal form of selfreference with the non-selfpenetrating singularity (Rapoport, 2020), which implicitly stands for a separation of the agent and the environment (Sengupta et al, 2006; Tozzi et al, 2007; Tozzi & Peters,2007).

This is framed ommitting /decrying the Moebius strip produced by antipodal dentifications in the setting of the Borsuk-Ulam theorem on spheres, on which they base their toroidal topological theory of consciousness and the neurosciences –and dual logic, implicitly.

Remarkably, it is known that the hippocampus appears to support the generation of a spatiotemporal mapping to the effect of navigation which then is a geometrical form.The geometry of motion that is produced under the given finite boundary conditions is that of a projective structure on a plane, which by the antipodal identification along the path of motion naturally produces the Moebius strip; see fig.6 . Remarkably, McNaughton et al acknowledge this projective structure in drawing, but refrained to mention that the topology that arises from this navigation system is that of the Moebius strip, and still point out, mistakenly, to the torus. The same mistake is at the core of Tozzi et al's topodynamics of the (metastable) brain. It is repeated in (Finkelstein et al, 2015) upon discussing the 180º symmetry of the bat brain spherical navigating system. There is one exceptional exception which does not commit this mistake, and which all these authors ignore completely: Roger Shepard's theory of psycho-physical complementarity (Shepard 1981,1982; Carlton & Shepard, 1990).

In the toroidal brain topodynamics, the censorship extends further to the torsion geometry of morphogenesis (Rapoport,2011b, 2013,2016I, III). Furthermore, it proposes an identification of consciousness with a metric-derived Levi-Civita geometrical connection (Tozzi et al,2017; Sengupta et al,2016) -as in GR- which by default has null torsion and is *free of selfreference*! (Rapoport, 2013,2016 I, II).The incoherence is most notorious in identifying the Organizational Architecture (OA) of Fingelkurts et al, with this model (Tozzi, Peters, Fingelkurts &Marijuan, 2017); however, the OA is based on the notion that that the brain-mind and particularly the self-consciousness concept operates "on the brain default mode network (DMN) that constitutes the neural signature of self-referential processes" for which there is significant empirical evidence,and which supports "…much of the mental content underlying self-referential mentation or thought and form a coherent framework of a sense of self"(Fingelkurts et al, 2016). On the brain side of the brain-mind operation in this model, it is the electromagnetic fields conceived as producing a nested hierarchy, ultimately manifesting to the mind through the waves reproduced in electro-encephalograms (EEG), which support different functional-cognitive states, which appear to support a nested homology of brain and mind vis-a-vis the EEGs organization. EEGs waves have a piecewise stationary structure which could be presented as a sequence of collated stationary processes . Each of them represents a building block of cortical activity, a steady, transient and self-organised operational unit which arises from empirical findings. They stand for a basic syntax of the brain oscillations, as a set of principles that allows the generation of rich combinations from a limited number of elements using a minimal number of rules. Actually, a relation between the rhythms of the brain syntax and that of language is envisaged (Buszsáki,2019). However, there is considerable influence between the different scale oscillations which is described as "the hierarchical nature of cross-frequency interactions may reflect a mechanism of syntactical organization" (Buszaki and Watson, 2012). However, *cross*-frequency interaction is the signature of non-hierarchical order, which is identified with the syntactic organization by which the brain can integrate many distributed local processes into globally ordered states. This cross-influence is at the basis of the brain as a

metastable system (Buzsáki, 2019) and its behaviour is signed by the 2:1 harmonics embodied either as the Moebius strip or the Klein Bottle, namely to 360º longitudinal rotation to return to the same point, supporting thus a topological protoform of Newton's third law, and the 4π symmetry of spinors (Rapoport,2013).To wit:

The structure of the EEGs wave frequencies already show a Moebius strip and/or Klein Bottle metaform: Indeed they have an architecture in which they progress linearly and free of gaps in a natural logarithmic manner (Pentonnen et al, 2003). Furthermore, in the linear representation the center frequencies of the EEGs bands follow a 2:1 harmonics (Pletzer et al,2010). This progression is found at the cortical histology, so that the distance between the branching of binary dendritic trees follows the recursive 2:1 harmonics, i.e. the octave progression, 1:2:4:8… (Bok, 1959). Also the Weber-Fechner law of the psycho-physical relationship between stimulus and perception is logarithmic, and rather than continuous it arises as a discontinuous stepfunction rising in octaves like those of a musical scale; see (Geissle, 1997). The former duplication is still crucial to the establishment of working memory, and its range as associated to the Golden Mean –characterizing the resting state of EEGs, which is conceived as the basis for information coding by the brain (Weiss & Weiss,2003) operating as a network with a Bose-Einstein statistics. It shows in the Tritone Paradox in music perception (Shepard,1964; Rapoport,2013), and still the built-in Klein Bottle invariance of the visual receptive Gabor fields basic to holography (Tanaka,1997) .

Rather than a nested Matrushka-like hierarchy, these octave-like architectures morph as a superposition of Moebius strips as in the cortical histology just mentioned,that of muscles and organs (Bell Pettigrew,1908; Thompson, 1917,1992), the mammal heart and its Möbius strip and spiral fibers torsion anatomy-physiology (Buckberg et al,2018), or still the lemniscal 1d projection of the Klein Bottle as the anatomy of the connective tissue crucial to mind-body integration (Rapoport,2016I). The concatenation of this harmonic octave architecture would hardly be conceived as nested, as much as a piano would be. For instance, when passing to the level of chords, rather than single keys, then the Klein Bottle appears as the psycho-physical metaform (Tymozcko, 2006). Returning to single keys, Shepard spells it as:

"Rectilinear scales of pitch can account for the similarity of tones close together in frequency but not for the heightened relations at special intervals, such as the octave or perfect fifth, that arise when the tones are interpreted musically. Increasingly adequate accounts of musical pitch are provided by increasingly generalized, geometrically regular helical structures: a simple helix, a double helix,and a double helix wound around a torus in four dimensions or around a higher order helical cylinder" (Shepard, 1982) Furthermore: "Just as continuous signals of speech are perceptually mapped into discrete internal representations of phonemes or syllables, continuous signals of music are perceptually mapped into discrete internal representations of tones and chords; just as each speech sound can be characterized by a small number of distinctive features, each musical tone can be characterized by a small number ofperceptual dimensions of pitch, loudness,timbre, vibrato, tremolo, attack, decay, duration, and spatial location. In addition,much as the internal representations of speech sounds a re organized into higher level internal representations of meaningful words,phrases, and sentences, the internal representations of musical tones and chords are organized into higher level internal representations of meaningful melodies, progressions, and cadences" (ibid.,p333-4). The brain-mind does appear to perform a dimensional reduction, actually to dimension two, as in the Klein Bottle surface (Rapoport,2013, and references therein), and Shepard focused on pitch. He remarked that this is about psycho-physics and ultimately, rather than the physical variables themselves, the perceived relations between the various tones within an octave do indeed depend on the context-induced musical key with respect to which those tones are interpreted.

The solution first proposed by Shepard was an helix as if wrapped on a cylinder which has for cross-section, the chromatic circle, in which the tones of an octave are placed, and lifted along the helix as placed on a vertical line projecting to the chromatic circle, each step being an octave above the prior chromatic circle, altogether producing an imaginal cylinder.Yet, since the circle of fifths also have an important role,this led to replace the chromatic circle by the circle of fifths being obtainable from it simply by interchanging every other point with its diametrically opposite point around the circle.Thus the single helix becomes a double helix following such a substitution (see fig 3c, ibid). An alternative construction replaces the double helix by considering each circle separately and thus together their Cartesian product forms a 4d-torus (Shepard, 1992; Brower, 2008). But now, on this 4d-torus the helical paths wind on producing a closed knot, very much alike the closed knots on a 2-torus edge a Moebius strip (Avrin, 2011, 2012). These closed knots on the 4-torus, known as the Villarceau circles (Berger, 2009), edge a Moebius strip, which are very much alike the Anatomy Trains of fascia formed by the connective tissue body of form; see references in (Rapoport & Perez,2018). Yet, the 4-torus is claimed to follow from the Borsuk-Ulam theorem of the toroidal topodynamics (Tozzi et al, 2016).

The bottomline is that (Western) music's diatonic space is a plural metamorphic being, as molecules in Topological Chemistry are: Diatonic space when arranged as a chain of interlocking fifths and thirds *is shaped as* a Moebius strip which further admits being rearranged as a cylinder and still nested structures (Brower,2008). Its embodiment elicit image-schemas such as CONTAIN of dual logo-physics (Rapoport,2014b b, 2016), and verticality associated to the feelings of tension and relaxation as we move up and down within its space by fifths or alternating major and minor thirds: Diatonic space appears as if an elastic psycho-physical space resembling a gravitational field -after Florensky, a *tonal field* (Korsakova,2019, and references therein) that may 'trap' or elicit light, as darkening or brightness.

Would we examine further the brain-mind as a metastable chaotic system in which the EEGs represent the stable functioning, then the chaotic attractors which represent the 2:1 arms of the Hopf bifurcation say as in the logistic map as a model of elicit Moebius strips and Klein Bottles, typical of chaotic attractors such as Lorenz, Rossler, etc. (Abraham & Shaw,1992; Crawford & Omohundro, 1984; Mindlin & Solari, 1997; Gilmore & Lefranc, 2011; Meiske & Schneider, 1987;Krauskopf & Osinga, 1999; Letelier & Rossler,2006). Theoretical and empirical studies appear to indicate that the dynamical *multistability* of neural circuits, as a transition between the chaotic attractors of the non-linear dynamics might underlie the switching between different perceptions or behaviours and can give rise to transitions between different oscillatory states of brain dynamic (Freyer et al,2011). Thus the nonlinear metastable dynamics of the brain-mind produced by the cross-influence between the EEGs octave rhythmic progression manifests as a sequential transition between non-orientable attractors, and still provides the basis for the self-organized control of a "communication-through-coherence" (Battaglia et el, 2011). However, the metastable behaviour from which the sequential chaotic attractors appear is conceived as a single 'complex' attractor, where the complexity refers to the shape of the manifold drawn by the trajectory, which can be characterized as a set of connected submanifolds, each capable of sequestering the trajectory for a limited period of time (Friston, 1997). These submanifolds are exactly the same as a normal attractor manifold but for the fact that they are connected to, or embedded in,a larger surface. At some point the trajectory will find this connection and a new transient will emerge as the trajectory moves off to another submanifold.

Remarkably, the non-orientable topology of chaotic attractors was not mentioned at all in the toroidal Topological Neurosciences of Tozzi & Peters. Nor the shape of the human bauplan, as

well as of all metazoans, i.e. species having a mouth-anus canal, is a 2-torus, however as a double covering space of the Klein Bottle (Rapoport,2014a). This conception is tantamount to reduce the body-mind to the brain, disconsidering all the previous elements and the role of torsion fields as the crucial morphomecanics of development, and neglecting the supradual ontoepistemology altogether for a banal form of selfreference embodied as the 2-torus, a geometry which ommits selfreference by default and still the underlying supraduality of chaotic attractors of non-linear systems (Rapoport, 2014b,2016 I) . Implicitly of this reductionism, it is as if the brain could be curtailed from its own bodily connected environment and the history of its development erased, and as if the mind-body could not communicate through itself nor participate as a semiotic system with others. For pioneering work of a lifetime on psycho-physics which shows otherwise, see (Grinberg-Zylberbaum,1981) and references therein. Remarkably the relation between criticality-metastability and the trascendence of dual logic was acknowledged by Bettinger; see (Bettinger,2017).

This neglect of torsion geometries is quite remarkable since Tozzi & Peters claim that "topological defects might be seen as a general mechanism in biology and neuroscience" (Tozzi et al, 2017). It is the case that the theory of topological defects of condensed matter physics is wholly uncompatible with Riemannian spacetime geometry and the associated Levi-Civita connection as in GR, but wholly related to torsion geometry (Kleinert,2008) and non-orientability.

Already in (Rapoport & Perez, 2018) it was shown that the $\boldsymbol{\pi}$-periodic hypercolumnar architecture of the primary visual cortex is related to its Klein Bottle topology, an architecture which follows from selforganization. This primacy of topological defects as supporting a topological organization for local density control has been found to be the case of cultured neural progenitor cells. They appear as a collective of cells organized as a nematic liquid crystal (Kawaguchi et al,2017), an example quoted in (Tozzi et al, 2017) to make the case of their toroidal principle for the brain. Liquid crystals are primal examples of topological ordering in which topological defects with asssociated non-orientable structures are generic (Bouligand, 1974), the brain matter, DNA and coherent water domains being paramount examples for their importance to life and particularly metabolism (Bouligand,1974;Mermin, 1979; Rapoport, 2016 III). The case is that nematic liquid crystals are non-orientable topologies, and so is the case of cholesteric and smectic crystals (Machon,2016).

7. THE KLEIN-BOTTLE, THE (META)FORM WHICH CONNECTS AND GENERATES: THE PRINCIPLE OF RESONANCE AND COHERENCE

We have proposed that the brain connectome harmonics patterns (Atasoy et al, 2018) have for ultimate support the Klein Bottle as the metaform of pattern recognition and pattern formation, i.e. morphogenesis. We offered that this is supported by the representation of the Klein Bottle as a standing wave in 5d to the effect of unraveling the 2:1 harmonics by removing the self-penetration of the Klein Bottle (Rapoport,2013; Rapoport & Perez, 2018).

Furthermore, the Klein Bottle and Moebius strips appear both theoretically and empirically, as the metaforms which allow to transfer energy as a resonant coupling between vibrating systems (Xu et al, 2016). This is so at some exceptional points of the spectrum of non-hermitean and hermitean systems, corresponding to closed and open systems, respectively, say the brain-mind, for the latter (Keck et al, 2003). These are degenerate states, i.e. they share the same eigenvalues, so that the coupling reflects a coalescence proper of a supradual logophysics. This is particularly so, for the non-Hermitean Cognition Operator given by the torsion of cognitive space,embodied by the Klein Bottle, which is nilpotent, i.e. its square is the Null Operator (Stern, 2000; Rapoport, 2011a,d). Its eigenstates are given by twistor fields given by quaternionic light rays as a torsion geometry of spacetime (Rapoport, 2009, 2011d). In this, we find, a

crucial instance of the already mentioned supradual logophysics, in which the selfreferential nature of light which is not seen but experienced as the seeing process, partakes both in the physical world and that of the brain-mind as represented by the Matrix Logic form of the Klein Bottle. The transformation of one eigenstate to the other of the coalescent state, as multiples of the 180º rotation of the Moebius strip, an antipodal identification which we shall retrieve all along the sequel, as a geometric-topological (so-called Berry phase) transformation which is crucial to the primary visual cortex $\boldsymbol{\pi}$-periodic architecture, and is given by the Klein Bottle topology of the topographic visual representation on this architecture to be discussed below (Rapoport & Perez, 2018) –for the Berry-phase interpretation see (Marcer & Rowlands, 2017). So, it is natural to speculate that the Klein Bottle metaform plays an additional role in ensuring the mind-body coherence and synchronization as the 40 Hertz oscillation (Buszáki, 2011) . The latter has been suggested to reflect the hexagonal Moebius strip architecture of the primal visual and somatosensory cortices, whenever the hypercolumnar architecture is diffuse -and which in the cases that the hypercolumn structure is notorious elicit the Klein Bottle morphogenesis of the primal visual and cortical topographic representations of the sensorium (Wright et al, 2014). Yet, given the non-orientable structure of the quantum fluctuations of the vacuum ('t Hooft, 2018), we propose that the resonance phenomenon pervasive to the whole cosmos and still its relation with holography is supported by the Klein Bottle metaform. Both resonance and holography appear already in the oscillations of the Sun and other pulsars (Kotov et al, 2020). Moreover, the Kozyrev torsion field coupling of the in-formation of a star along its past, present and future location (Kozyrev,1971; Lavrentiev et al, 1991) -which we have argued to be the case of the synchronization producing stereoscopic vision from the asymmetry of each eye's vision (Rapoport, 2010b), the "mimesis" of double-star systems, or still the palindromic Moebius strip structure of the histograms of random phenomena that appears recursively in cycles of 24 hours, lunar months and sidereal years as if a checksum algorithm of harmonics would be operating to produce them (Shnoll, 1990).

Paraphrasing Gregory Bateson, the Klein Bottle is the (meta)form which connects, and further supports morphogenesis, meaning and pattern formation, semiosis, cognition and recognition.

8. KLEIN BOTTLE LOGOPHYSICS, THE CONTAIN IMAGE-SCHEMA, EMOTICS, REFLEXIVE MONISM AND THE CONSTRUAL OF THE WORLD

For the Klein Bottle logophysics , or still Velmans' Reflexive Monism to be introduced below,for which the physical space of the world actually is a projection of space-as-experienced, it is the situatedness what actually is construed as prior to any other content of experience. Situatedness is essentially attached either to an inhomogeneity or a singularity , which is not the absence of inhomogeneity but rather the situatedness of a distinction. However, as in the physical space as the space of experience, this situatedness is dynamic, as also Heidegger conceived it. However, this situatedness is a manifestation of temporality, time-space as Heidegger conceived it in his work *Time-Space*. Time as the operationality on space.

The current conception of space is tied to the dual logophysics for which the true and false categorization is secondary to situatedness and takes the form of Outside and Inside, or the converse,vis-à-vis a boundary. Remarkably, several authors have contested the hegemonic dual categorization into Inside and Outside, given a boundary. Already Wiitgenstein alerted to the reductionism implied in its positing as a recurrent image-schema. Merleau Ponty referred to it in the following terms: 'The world is wholly inside and I am wholly outside myself" to which Malpas commented: " at the same time suggests a breakdown in the very dichotomy that is invoked. The stuff of our 'inner' lives is thus to be found in the exterior spaces or places

in which we dwell, while those same spaces and places are themselves incorporated 'within' us", (Merleau Ponty,2013, p. 474) a Klein Bottle embodiment projected to construe space as the locus of our affections, or that of the ratio of our thinking. Bachelard retook this issue in his *Poetics of Space* placing the experience of space as the meaningful locus for our affections, very much on the path of Marcel Proust for whom time recurs on space together with our memories and affections (Malpas,1999,2007). For Bachelard, who criticized the hegemony of the Out-side/Inside dicotomic duality and its ontological claim of primality," the life of the mind is given form in the places and spaces in which human beings dwell and those places themselves shape and influence human memories, feelings and thoughts. In this way, the spaces of inner and outer – of mind and world – are transformed one into the other as inner space is externalised and outer space brought within"(Malpas,1999, p5). Thus, rather than adopting the Inside /Outside dichotomic ontology, we can translate Bachelard`s intuitions:The world is wholly insi-de and I am wholly outside myself considered that mind and world operate as a Klein Bottle logophysics in which Inside and Outside intertransform. He was not the only one in the critique to the Inside/ Outside dichotomy, a critique prompted also by Merleau Ponty. Without iden-tifying the Klein Bottle he expressed as it as: "The world is wholly inside and I am wholly out-side myself' (Merleau Ponty,2013). We can find an homology in the relational theory of reflexive sociology: "the body is in the social world but the social world is in the body" (Bourdieu,1982), the latter being a core principle for epigenetics (Jablonka,1995) and phenomenologically elaborated in sociology in terms of self and hetero-reference and their projection and retrojection cycles, viz. the Hyper Klein Bottles, as by (Berger and Luckmann,1966).

However, the phenomenology of Merleau Ponty returns to the issue of time, in which events occur in a space which cannot be circumscribed to the physics of it. Time spatializing in a way such it transcends the space of experience, which becomes the locus for memories of emotions, events, objects, loved ones, society at large. This time which at each point the subject evokes the past returning to it imaginarywise, constructs a present and is already projecting into the future. This time, Proust described as an open space which any particular life in remembrance takes for its own. Contrarily to the space of experience which appears as bounded by the horizon and continuing to the sky vault, this time which is fully extant at any given locus, is nonlinear, rather than linearly extended. Consciousness through memory associates it to a virtual return in time which is experienced as real emoticswise, an imaginal displacement of selfreferential nature, and still projects itself selfreferentially to a prospective future, yet as a search for alterity in doing so. Thus, time conforms in its potentiality and as an operation of consciousness travelling-through-memory-of-the-past-and-prospection-to-the-future, producing imaginal closed loops to both the past and the future. Thus time unravels from the present to the past and the future, conforming a three dimensional unity -rather than one-dimensional sequentiation for the description of change. Thus, it has been considered as a signature of consciousness supported by selfreference, and the very basis for higher order thought such as art, science, technology and culture at large: without this displacement and prospective exploration of the self, problem solving, decision making, creativity, selfreflection would not be. At the unconscious level it is fully supported by the myriad of integrated closed loops of the organism and its subsystems, as already discussed, and also the closed cycles of the cosmos to which our bodies appear to be synchronized to. At Heidelberg's late and largely unnoticed conference *On Time and Being* he called it as time-space. Space as experienced, what appears as the world, is supported by this operational non-linear three-dimensional time: thus, time-space. Astrophysicist Kozyrev came to concur independently with this conception of

an active time-operator (Kozyrev, 1971) whose non-linearity is clearly manifested at the boundaries of their propagation (Rapoport & Perez,2018).

The location of the perceived world has been a controversial issue. The usual take is that we represent the world in our brains, so the representation is a matter of insidedness of an inherently exterior word. Turning Outside-In and Inside-out the usual take, Lehar offered the idea that what we see as exterior is our brain image contained in our skull (Lehar,2003), which anyway, since there is no absolute cognitive framework, nor this nor the usual take can be nor assessed nor falsified for. Remarkably, for the primitive humans there was no distinction of the inside and the outside as manifested in the cave art (Lewis-Williams,2004). In fact, the Charles Bonnet syndrome shows that even in the case of visual impairment, complex visual imagery though reduced in size, appears to be produced as if outside, even in the full absence of stimuli of the eye. The ancient process of incubation in caves under complete deprivation of light, which has been currently reproduced, shows the same complex imagery appears as if projected on an exterior screen (Kingsley,1999) . Reporting this imagery of blind patients producing moving complex imagery seen as if exterior, Oliver Sacks put it: "we see with the eyes but we see with the brains. The latter is often called imagination…the inscapes…Hallucination are different in that they seem to be not of our creation, nor our control. They seem to come from the outside, and to mimic perception…alike films, but unrelated to the person " (Sacks,2016). The current controversy can be resumed as follows. There is the notion that the percepts are formed in the brain, say vision, and correspond to representations of exterior objects; a naïve realism. A different take is that the perceptions of our phenomenal experience appear as if projected from our brains to the exterior world very much as if a projector would do the job. The phenomenal world of experience appears as if spatially located producing an exterior world (Velmans,2009).

With the exteroperceptive system, say auditory, olfactory or vision we receive stimuli from the exterior world which is further used to create those contents of consciousness which correspond to the event and objects that produce the experienced world of space and time. With the interoceptive system our body experiences interoceptive sensations, including kinaesthesis, and bodily pleasure and pain. Emotions however transcend the Inside and Outside dual categorization since they may include both interoceptive body sensations with cognitive elaboration of exterior data. Thus emotions in conjoining exterior and interior experience already point out that the contents of consciousness, in these case emotions, surmount the dual Inside and Outside ontoepistemology. Being so, emotions provide the grounding for a fundamental property of consciousness as understood by Panskepp and Damasio, both empirical neuroscientists looking for the neural correlates of emotions. Still afar from the notion of ecological integration through the Klein Bottle logophysics and its unity-in-diversity, in the Information Integration Theory of Consciousness, is claimed: "According to the theory,consciousness corresponds to the capacity of a system to integrate information. This claim is motivated by two key phenomenological properties of consciousness: differentiation—the availability of a very large number of conscious experiences; and integration—the unity of each such experience" (Tononi,2004). The body-mind indeed integrates the information of emotics,which clearly have no discrete character: emotions are fully integrated, per se, though at times they might be experienced as localized in the body (say, rage, associated to the liver) and still the diversity and richness of emotions as an intertransforming palette of the self, the bodily owner of them. Indeed, emotions are the matter of Firstness, the domain where the principles of classical dual logic are invalid, as already discussed. Here, digital information, as in the usual theory of information by Shannon, cannot be the case. Remarkably, Buzsáki has made the point that the ascription to an outside-in representation for the operation of the brain misses

the more recent revelation of the brain as a maker, an inside-out transformation (Buzsáki, 2019). After all, the very evolutionary history of the nervous system is that of an introversion of the Exterior surface of the ovum which later turns inside-out. In this take, language is based on internalized action, and social action is action, heterarchy ultimately resolvable to a recursive extension of the Klein Bottle intertransforming Outside and Inside. The mirror neuron phenomenology being the case in point, in which Other and Self are mutually constitutive and generating. Or still, language and the role of metaphorization (Rapoport, 2014b). Thus, the ascription to the brain as acting inferentially, as a Bayesian operator (Buzsáki, 2019), for which a Markov blanket (MB) is proposed (Friston,2010,2012). The MB "consists in two sets ('sensory' and 'active' states) which influence each other in a circular fashion: [active] external states [Outside-Outside; Klein Bottle-wise, as in the other states of the blanket] cause sensory states which influence – but are not influenced by – internal states [Outside-Inside], while internal states [Inside-Inside] cause active states which influence – but are not influenced by – external [Inside-Outside] states " (Solms and Friston,2018). These identifications seem all to natural. Rather than a metaphor, the somatosensory system is a MB, supporting the unity of action and perception, the Klein bottle its topology (Werner,1971).

Also the imaginal seemingly visual hallucinogenic experience such as the one elicited by the Bonnet syndrome point out that this construal of seemingly exterior images point out also, as in the case of emotics and particularly in the construal of either objects or events, that consciousness surmounts the dual Inside Outside divide. In the case of hallucinatory experiences say induced by entheogens in which the images are felt and visualized as if more real that the visual experience of usual perception, and at times a metamorphosis of them still fusing with the latter, are construed as internal images turning to exterior ones. It is somewhat compelling to qualify the experience of both the Bonnet syndrome and the hallucinatory instances as manifestations of the Klein Bottle logophysics transitions Inside/Inside, Inside/Outside, Outside /Inside and Outside/Outside. Already inner experiences such as thoughts, memories and so on normally consist of verbal, visual and other forms of imagery. Some experiences derive from a combination of resources and thus our body image which fuse external or surface visual imagery with interoceptive information such as body sensations constitutes a surmountal of the dualistic divide. In other words, body image and emotics already point out to a Klein Bottle logophysics, rather than a dual one. Velmans already noticed that the con-tents of consciousness cannot be considered as contained in three-dimensional space, according to the CONTAIN image-schema, or still as fullfilling the formula of objects-in-space-before-subject (Rosen, 2006), they constitute the very construal of the world-as-experienced.

Velmans:" the contents of normal phenomenal consciousness are neither beyond three-dimensional space (as dualists assume) nor contained within just a tiny bit of three-dimensional space (as materialists assume). Rather, these contents define and fill three-dimensional space as they are none other than the everyday world, or universe, as expe-rienced. What one experiences at a given moment depends, of course, on how one directs one's attention. Conscious contents differ enormously, for example, if one's eyes are open or closed. However, with open eyes, the contents of consciousness stretch to one's visual horizons. They include not just inner and body experiences, but also the external phenomenal world that we conventionally think of as the 'physical world'" (Velmans, p.294).

What a Klein Bottle logophysics does -additionally to support consciousness- is to operate contextually. This is so in the sense that it expresses and manifests the embedding relationship vis-à-vis the cosmos and the very coherence that is realized by the fact that, as a matter of principle, the cognizing agent both through consciousness and the largely unconscious operations that support it, so it is a full body and the cosmos at large, operate through the same

supradual logophysical principles. Consciousness is neither restricted to the brain nor to the supporting body, but it exists through the non isolation of them vis-à-vis the cosmos, an integration realized and supported by a cohering principle which does not separate outside and inside, relative to the individual, and the cosmos at large. In this regard, consciousness resembles light, which rather than seen is seeing –i.e. experienced- and crucial to the whole body coherence through the connective tissue, and thinking. as we already discussed its relation to supraduality and cognition, and space time torsion geometry of Penrose twistors (Rapoport,2011b).

Velmans unaware of the Klein Bottle logophysics, expressed it in his Reflexive Monism thus: "…human consciousness is embedded in and supported by the greater universe.... The contents of human consciousness are also a natural expression or manifestation of the *embedding* universe [our emphasis]. In humans, then proximal causes of consciousness are to be found in the human brain, but it is a mistake to think of the brain as an isolated system. Its existence as a material system depends totally on its supporting surrounds, and the contents of consciousness that it, in turn, supports arise from a reflexive interaction of perceptual processing with entities, events and processes in the surrounding world, body and the mind/brain itself" (ibid., p.295-6).

Still: "Reflexive monism (RM) suggests a way of understanding these relationships that neither splits the universe into two incommensurable mental and physical substances nor requires consciousness to be anything other than it seems. It neither splits consciousness from matter nor reduces it to a state of the brain. Instead, it suggests a seamless, psychophysical universe, of which we are an integral part, which can be known in two fundamentally different ways. Whether one adopts the perspective of an 'external observer' or a 'subject', the embedding surround, interacting with brain-based perceptual and cognitive systems, provides the supporting vehicle for one's conscious view, and what we normally think of as the phenomenal 'physical world' constitutes that view. Nor does reflexive monism ultimately separate the observer from the observed. In a reflexive universe, humans are differentiated parts of an embedding wholeness (the universe itself) that, reflexively, have a conscious view of both that embedding surround and the differentiated parts they think of as themselves." (ibid, p.327-8).

Other investigators have raised similar ideas." Instead of perception depending largely on signals coming into the brain from the outside world, it depends as much, if not more, on perceptual predictions flowing in the opposite direction. We don't just passively perceive the world, we actively generate it. The world we experience comes as much, if not more, from the inside out as from the outside in" (Lloyd, 2017). Remarkably as the latter observation is to make the case of a mindlike cosmos, performing digital computation, this points out to a cosmos in which inasmuch as the dichotomy Inside/Outside is not valid, so is the case of digital/analogic dichotomy. In other words, the experienced world rather than being reducible to an Outside/Inside categorization in which we passively construct a representation from Outside data elaboration, which is resolved as perception, it is a Klein Bottle unity which is predominantly Outside-Inside and Inside-Outside generated, i.e. through the intermediation states related to the selfpenetration of the Klein Bottle. The difference with an hallucination conceived as an uncontrolled perception, the usual perception construed by the Klein Bottle integration is such that the Inside/Outside is somewhat subjugated to the Outside/Inside elaboration of the seemingly Outside world of experience. In this more general sense, usual perception is an hallucinatory state, but one such that the Outside world data places a commanding orientation of construal. This experience of subjugation to alterity, that is common in certain altered states of consciousness induced by entheogens, are accompanied by expe-

riences of metamorphosis which elicit the supradual nature of the logophysics underlying the experience. However, in the relation to the Other, the living of Alterity, does not require necessarily subjugation, but rather an ethics which the philosopher Emanuel Levinas characterized as follows. For Levinas "the autonomous "subject" is constituted as the continual encounter with alterity that renders it topologically not as a unity but as Klein Bottle. "Subjectivity realizes the impossible exigencies –the astonishing feat of containing more than it is possible to contain "(Levinas, 1979). However, this relation to Alterity does not place any distinctions, nor on the side of the subject nor of the Other. This is the case of the HyperKlein Bottles in which the self and the Others as pluralities are interwoven as an arboreal organism which in distinction with the rhizome due to Deleuze, is not necessarily hierarchical but heterarchical as well, nor unique in structure and form. They are centerless, heterogeneous, both static and everchanging networks of Deleuze's rhizome with an important feature: the selfpenetration and heteropenetrations which as a tree rhizomes seemingly have not. It is not only the living body that harbours such a rhizome, but ecological and social systems as well. Remarkably, Velmans' Reflexive Monism extends to the construal of our environment, in its full cultural, technological, institutional, economical, architecture, religious and value-system terms (Berger and Luckmann,1966). This ultimate projection is such that intersubjectivity is the relational mode of construal of knowledge and the manifold value systems, in particular those of verity values.

8. 1 Klein Bottle Supraduality, Torsion and Quantum Mechanics

The plurality of verity values that naturally arise with the Hyper Klein Bottle supradual logophysics is very much alike Quantum Mechanics, for which the logic associated to measurements is a lattice operating with a non-Boolean logic, but for a particular context a Boolean logic turns out to be the case. Each culture sets up a context, at times uncoherently vis-à-vis the others. Events in Quantum Mechanics are contextual (Haven and Khrennikov,2013) and the particle-wave complementarity and more fundamentally quantum entanglement are supported by non-Boolean logic (Jaeger, 2016). As for Quantum Mechanics and its role in cognition, the decision process itself is supported by non-Boolean logic (Roy,2017) and the underlying principle is the non-commutativity of quantum operators,

Non-commutativity turned into commutativity as if the Klein Bottle would be two-sided appears to play a crucial role in the cognitive representation in schizophrenia, following the studies of the latter by Matte Blanco; see (Rapoport & Perez, 2018) and references therein.

8. 2 Klein Bottle Supraduality, Topological Entanglement, Torsion, Light Rays, Quantum Jumps and Non-commutativity

Thus, in the setting of the Matrix Logic form of the Klein Bottle logic (Stern, 2000,2014; Rapoport, 2009,2011a,d) the non-commutativity which sustains the decision process and Quantum Mechanics at large is encoded is the non-duality of True and False operator. It is supported by the Klein Bottle as its resolution through its non-null torsion commutator. At the level of spacetime, torsion is associated to the anticommutativity of the clockwise and anticlockwise directions of the dislocated infinitesimal parallelogram produced by torsion associated to the enclosure of a singularity, the torsion itself produced the closure , or still, the anticommutativity of the torsion tensor or of the torsion two-form *(*Wikipedia. Torsion). In the case of the torsion propagating waves associated to quaternionic light rays, particles are lower-dimensional components of the wavefronts (Rapoport,2009, 2011d).

Light rays are the actual support for experience, which is very much anchored in the visual mode. Furthermore, the electromagnetic field as the carrier of photons plays a crucial role in

Quantum Mechanics. Would we consider spacetime wave propagation which further is constrained to propagate as light rays –the eikonal equation of geometrical optics, as nilpotent fields- in its longitudinal manifestation, we found that they produce a torsion field given by the spacetime gradient (or differential, in the sense of calculus) field of the complex logarithm of these waves, which their nodes are loci for quantum jumps (Rapoport,2009,2010a).So on the one-hand, quantum jumps which are the most basic phenomenae to the making of reality as the primeval differences creating differences, emerge from torsion fields propagating as light rays. Associated to this torsion differential of the complex logarithm of the wave functions we find an Aharonov-Bohm potential supporting a phenomena of coherence, crucial to life. As for quantum jumps, they produce discontinuities in the energy-momentum tensor leading to the existence of a cosmological time in a *global* canonical decomposition of spacetime, salvaging the underdetermination of the Cauchy problem for the Lorentzian metric (Maschkevich, 2004).

Would we extend the numerical field to the quaternions, rather than the complex numbers, then the importance of light becomes still more crucial. On the one hand since the phase of this light rays are three dimensional, actually valued on the unit 3-sphere, then phenomena such as phase modulation appears to be the case. It supports anticipative behaviour (Rapoport, 2010b,2013, 2016 III) and still allows for non-orientable topology of the wave polarization (Freund,2010) which we have argued as crucial to biophotonics and DNA coherence and still its interactions with coherent water domains crucial to metabolism (Rapoport, 2016 III).

Remarkably the complex logarithm function which produces the quantum jumps as spacetime events and trigger the elementary visual data, is further introjected as the retinotopic visual representation which at the level of the primary visual cortex is projected producing the visual topographic map on the hypercolumn architecture. Ultimately the topology of this map is the Klein Bottle, as discussed in (Rapoport,2013; Schwartz, 1971; Swindale,1996;Tanaka, 1995, 1997). Also, it appears as the dermatomal helical trajectories on the human body periphery supporting the topographic map of the somatosensory mode, where each point on the skin and all those below its surface position are projected to a single point on the primary somato-sensory cortex, where the topology of the map is, again, the Klein Bottle (Werner, 1970; Werner & Whitsel,1968).

Therefore the complex logarithmic function which, as in the physical realm, has singularities (in the physical case the loci for quantum jumps), which in the neurological case have for correlates vortical structures in the neurocortex (more specifically, the so called hypercolumnar structure); in the latter case, these singularities stand for the points in the neurocortex in which the stimuli orientation mapping given by the logarithmic map becomes multivalued. Yet, this map is the analytical representation of the 3*D* outer body (and its surface) to a unique 2*D* plane in the neurocortex in which each point codifies a whole hypercolumn vortex which is not anatomically distinguishable (Wright et al, 2014). In the present conception, it will transpire that in distinction to other approaches to consciousness, which claim additional noetic higher-dimensions for consciousness following the Einstein tradition of treating complexity through the introduction of additional dimensions, the dimension 2 common to the Klein-Bottle, the space of all cognitive states in Matrix Logic, the undistinguished plane from which Matrix Logic arises from the primeval distinction, the dimension of the phase space associated to will-self-reference-control, the dimension for holography in general and in particular in the neurocortex, and the complex plane (or still, the Riemann sphere) for the representation of the complex logarithm, will be singled out (Rapoport,2011d).

Let us discuss the logophysical bearing of light, which is not seen but the seeing process field. Seeing involves quantum jumps, which already appear with the impinging of a photon on the retina, a basic phenomena even if visual perception is associated to a visual system, rather

than the elementary sensate, as in Gibson's ecological mind. As the photon supporting the seeing process we find a fusion of Kant's nouminal (the 'objective external' world) and phenomenal (the 'internal' perceptual) realms. Yet, there is much more than a logophysical identification of light rays as torsion fields and the seeing process which we shall describe in the sequel, which has to do with the Intelligence Code (Stern,2000;Rapoport,2011d). It has to do with the non-commutativity of quantum operators operating as cognitive operators, interchangeably, in Matrix Logic form of the Klein Bottle, which has Quantum Logic, Fuzzy and Boolean Logics for particular case. We notice that the matrix representation of the Klein Bottle, is the Hadamard matrix, the crucial logical gate of Quantum Logic and Information which is the single unique gate to transcend Boolean logic computations (Aharonov, 2003). Furthermore, it operates as a selfreferential cycle that projects the topological entanglement of the Klein Bottle to the classical true and false states as cognitive states of Boolean logic which by further recursion returns to the entanglement (Stern, 2000,2014; Rapoport, 2011, a,d). This is a decohering/entanglement-cohering cycle which is purely topological, and primal vis-a-vis quantum entanglement.

Actually, this topological entanglement supersedes the allegedly dualism of action and reaction of Newton's Third Law, both states corresponding to the topological entanglement, being a single *global* state with two *local manifestations*, the two *loca*l normal vectors to the non-orientable surfaces (Rapoport,2013,2016 I). No 'semiclassical' transition from quantum to classical physics is the case. Clearly topological entanglement as non-orientability is primal vis-a-vis quantum entanglement, or still Newton's third law, as a classical physics manifestation.

Due to the relation between the eigenstates of the Null Logical Operator (with all matrix elements being 0) of Matrix Logic with the twistor representations of the extended photon that decur from the quaternionic-valued light rays propagation as a spacetime torsion geometrodynamics, we have claimed that the *photon supports seeing-thinking*. This is because of the above mentioned relation with the cognitive operators of Matrix Logic, whose Null Operator we named the *Mind Apeiron* since its eigenstates support all the potential cognitive logical states (Rapoport, 2011d, sec.6). The Null Operator corresponds to the empty state of the Laws of Form. We have identified the primal distinction with the torsion field (Rapoport, 2011a,d, 2016 I), which in its twistor representation precisely factorizes the empty state. Matrix Logic associated to the Klein Bottle as its metaform, considers quantum states as cognition states interchangeably, the latter operated by non-Hermitean operators whose concatenated operations allow to intertransform the action of quantum operators for two-spin systems in the action of Hermitean cognition operators, and more fundamentally a topologic form of entanglement, just like Merrell conceived of semiosis (Merell,1996). Thus, seeing associated to twistor torsion light rays, and thinking, in the sense of these intertransformations, can be identified, and symmetrically at that. So, indeed a topological electromagnetic protoform of panexperientalism-panpsychism (Skrbna, 2005) is embodied in the seeing process as equated with the thinking process, in the memory supporting form of torsion as an asymmetry supported by twist (Leyton, 1992) and the non-orientability of the Klein Bottle.

In this setting the Klein Bottle is embodied as the torsion of cognitive space , as the non-commutativity of the True and False operators, which are non-dual (Rapoport, 2011a,d). This produces an *eigenform* (Kaufmann,2017) and still an identity between the action of the cognizing self-referential mind and the quantum action of spin. Thus the cognitive logical processes of the subject become related with the physical field of spin on the quantum states, supporting an interface between the in-formational and quantum realms, in which topology, torsion, logic and the quantum world operate jointly. Yet, due to fact that for the Klein-bottle there is no Inside nor Outside, the exchange can go in both ways, i.e. the quantum realm can

be incorporated into the classical cognitive dynamics, while the logical elements can take part in the quantum evolution (Stern,2000;Rapoport,2011d).

We have constructed the quaternions in terms of logical operators in Matrix Logic (Rapoport,2011a). So, we can represent this result on the torsion twistor representation built on terms of the quaternionic light rays as either an 'objective' space representation of the objective-subjective photon, or as a 'subjective' representation of it in terms of a quaternionic structure which stems from the laws of thought. Remarkably, from the quaternions in any of the two representation, the logical quaternions in Matrix Logic or the usual quaternions built by Hamilton as unrelated to logic, we can obtain some cosmological solutions. Indeed, the natural metric in the Lie group of the invertible quaternions, can be parametrized as the closed Friedmann-Lemaitre-Robertson-Walker metrics (Trifonov,2007) which constitute one of the most important classes of solutions of Einstein's equations and furthermore, as the Carmeli metric of Rotational Relativity (Hartnett, 2005).We recall that the latter was introduced to explain spiral galaxies rotation curves and 'dark matter'. We stress that these derivations do *not* require solving the Einstein's equations of GR but are intrinsic to the quaternions, or if wished, to Matrix Logic. This raises the question on what are we actually representing: Is it an 'outer' world, or a Klein-bottle cosmological fusion of the physical and the noetic realms, as the Klein Bottle ontoepistemology claims to be the case? We shall retake this upon examining Shepard's principle of complementarity.

Another form of putting out this is to conceive the Klein Bottle as the fusion of the opposite chirality Moebius strips that arise from the blowup of the origin –in the sense of Algebraic Geometry (Griffiths,1978), the null state of Matrix Logic, the undistinguished state in Laws of Form. The reader may compare this logophysics with the mythology of the Big-Bang,and its makeshift unprincipled ontological nature. In distinction with the blowup of the origin, it is deprived of selfreference, not to mention memory. Remarkably the holographic paradigm of theoretical physics proposes that all information is somewhat embodied on the two-dimensional surfaces of the event horizons of singularity "black holes", whose non-orientable topology was identified in the singularities of the Minkowski geometry of the vacuum quantum foam ('t Hooft,2018).

Actually, when the Renaissance Masters were looking through the pinwhole of a frame what they were unawaringly setting was not only the geometry for the appearance of the Self as a detached observer, supradually superseded by the antipodal identifications that produce non-orientability, but rather a unique ontopoiesis: the supradual re-creation of Self and Universe, jointly. Indeed, the blowup of the origin which upon identification of antipodal points of the ensuing projective space construes non-orientability but particularly also construes it in the kinematical geometry of Quantum Physics, the so-called Quantum Geometry (Bengtsson & Zyczkowski,2006) in which it is embodied. Particularly, the ensuing dynamics of quantum systems as characterized by the so-called open Schroedinger equation is such that the reduction of the system to a certain state is described by a torsion gradient field related to the expected value of the total energy (the hamiltonian of the system) and its variance (Rapoport, 2007). There are antecedents to this. Early in the 1980's it was proved that the formalism of Quantum Mechanics in terms of projectors can be derived from Boolean logic (Orlov,1978, 1984) and this was further extended to Clifford algebras, where an idempotent element, e3, basically stands for the Klein Bottle action on superposed states, but unacknowledged as such (Conte,2011) .

8. 3 Klein bottle logophysics and the Integration of Contextuality Through Constraints

Returning to the discussion of the construal of the experienced world, be that the physical and the imaginal domain, whatever the case may be, the upshot is that there is, in principle, no clear cut distinction to the subject to distinguish whether what is being experienced pertains to an experience of the world outside, an hallucination, a lucid dream, an altered state of consciousness.

The material system which is the living body depends on the surrounding space as an agency of the interaction of space, mind and body in a coherence. This coherence is autopietic –yet supradual, rather than the autopietic Outside-Inside duality purported by Varela-, selfreferential selfcreating selforganizing both in its physical, mental and physiological domains, all operating supradually. In the physical domain this already starts with the torsion selfreferential geometry of cycles with a non-orientable topology.

In Varela conception of autopoiesis an organism delimited by its boundary selforganizes in terms of the dual logophysics defined by the interior and exterior of the organism. The boundary operates as a dual logical gate, in which the boundary is semipermeable by fundamentally receiving matter and information somewhat passively unintegrated with its surroundings: this is a non-ecological conception. There is no integration with the environment, no action of the organism producing it, no codetermination. The organism does so by countering the growth of entropy due to its interaction with the environment and still the disorganizing action of the latter considered mainly as independent of ourselves, rather than integrated to ourselves in all cases. In this setting it is the operational closure of organism that sustains the autopoiesis. It operates autonomously but Varela does not mention the crucial importance of homeostasis to biological psychology as introduced by Panksepp (Panksepp, 2004).

Which topologies arise with the locative nature of our bodies-minds, integrating the developmental processes? Human bodies, actually all organisms having a mouth to anus canal, have a toroidal architecture, with the Klein Bottle and Moebius strip as sections of the torus, the latter being their double covering space. However, these organisms appear as $semiotics$ systems associated to an overall (Hyper)Klein Bottle integrity, whose projections are embodied in the diverse organismic systems. This is clearly the case of the human body, with its Moebius strip heart architecture as well as that of the lemniscal (Moebius strip projection on the plane) fascia connective system crucial to its integration made of luminous liquid crystals, i.e. torsion geometries, the cortical topographic maps of the visual and somatosensory sensoriums, etc. (Rapoport, 2013, 2016) and still of its genome (Rapoport, 2011b,2016 III). This already points out to the topology of our bodies arising through their locative nature vis-a-vis the environment, as associated to an organismic system of signs operating through a Klein Bottle logophysics as a semiotic in-formation process.

Yet, if the phenomena that we experience in the usual state of consciousness constitute what we think of as the everyday world then the natural question that remains is: The phenomenon that we experience in hallucinatory states, or in the states of sensory deprivation, what world do they constitute? As remarked by several authors what is omitted in the usual state of consciousness as its contents is contents of consciousness in a typical awake state include the external 'physical world' as-perceived along with various body and inner experiences. But they exclude a far greater set of entities, events and processes within the external world, body and mind. Given their close linkage to consciousness, it is of particular significance that the operations of the mind/ brain are largely nonconscious. Metaphorically, the contents of consciousness have often been likened to the tip of an iceberg. The bulk of the mind, like the iceberg, remains unseen below the water. The present analysis extends this metaphor. Once one expands consciousness to include the experienced body and surrounding

phenomenal world, what is 'above the waterline' is not just the tip of the iceberg but everything that one can experience extending to one's perceptual horizons. What is 'below the waterline' expands correspondingly to include the entire universe of entities, events and processes that, at a given moment, has no representation in what we experience.

9. SHEPARD'S PRINCIPLE OF PSYCHOPHYSICAL COMPLEMENTARITY AND THE REMARKABLE HOMOLOGY OF PHYSICAL SPACETIME DYNAMICS AND THE PERCEPTION OF MOTION

Returning to the issue of the interface of the physical exterior world and that of perception cognition as a complementarity which upon introjection appears to be related to the appearance of life and the very stability of organicism, some general considerations seem in order. While a principle of complementarity in terms of the Klein Bottle logophysics is warranted, merging objective elements of the material world with subjective elements which though directly referred to first-person operation intersubjectivity has already established the common validity with with others , in the case that the former objective elements are no longer, we are left to focus on what Shepard considers the primal problem of perception: understanding the subjective elements that make up the avowed complementarity. Indeed, as Shepard puts it: "… even when the external objects or events are fully stimulating the appropriate sensory surfaces, if the corresponding internal processes fail to occur—whether for reasons of physical disruption of the afferent neural pathway, cortical damage, an altered state of consciousness, or merely temporary inattention—there will ensue no corresponding perceptual experience. And, clearly, behavior is mediated by internal representational processes" (Shepard, 1982). We insist, at this constitutive level of perception, and integrated to action at that, we delucidate this through psychology, not, say, through quantum physics. Otherwise, it is a meaningless methodological pot-pourri, a work of fancy.

Here the role of constraints step in. These are usually neglected when dealing with the complementarity referred to the material world linked to space and time, the latter mainly as a measuring and situational parameter- thus spatializer, rather than an operator construing the complementarity. In this case the laws of this world are paramount to the determination of the forms of causality, which are forcefully reduced to physical causality, and we are left with the epistemology of positivism, very much hegemonic if not dominant in sciences and present in worldy affairs, no less. Laws of the exterior world are incomplete to this effect, without placing further contextual elements, though by default restricted to this somewhat reduced spatialization, since the latter, as we shall see apply as well to the principle of complementarity as exercised by the Klein Bottle logophysics, or even to the notion of a brain as a passive receptor of stimuli provided by the physical exterior world.

These contextual elements to harness the laws for reaching a concrete nature are the so-called boundary conditions which complete the abstract laws. *Mutatis mutandi,* would we turn the objectivist world turn inside out, an operation which though primal to the Klein Bottle complementarity, turning the internal representation the objects of this operation, the constraints appear to take the crucial role which now the science of cognition requires, rather than physics, chemistry, or more generally the sciences structured on terms that prioritize the hegemonic take. Whatever the prejudice we harbour for the 19th century as fully dominated by objectivism, not only psycho-physics was born but also some neat reflections of the modeling relation we are presently entertaining. Hertz's notion of the modeling relation which is presented in the introduction to The Principles of Mechanics (1894).

"We form for ourselves images or symbols [Scheinbilder] of the external objects; the manner in which we form them is such that the logically necessary [denknotwendigen]

consequences of the images in thought are invariably the images of materially necessary [naturnotwendigen] consequences of the corresponding objects. (...) Experience shows that the demand can be satisfied and that such correspondences do in fact exist". (Hertz 1894, pp. 323-324 [1956, p. 1-2]) (Quoted from Jon Umerez).

Thus expressed, what is being disconsidered is the adapting nature of the modeling relation, an adaptation which is crucial to survival, and its transformation to acquired traits in which the prior modelization has become embodied and thus subjected to inheritance. Our mental apparatus has been finessed by the evolution to allow our survival. Thus, the massive unconscious activity of the body-mind of which conscious awareness and emotics is the 'tip of the iceberg' is claimed to operate as Bayesian inferential systems an in terms of a minimal surprise principle that can be associated to a minimal effort, as a psycho-physics extension of the Maupertuis-Fermat principle of classical mechanics and geometrical optics (Friston,2010,2012).

Let us now pass to the consideration of the underlying principle that subtends cognition in terms of cognitive maps related to the representation of motion. As before, the principle of minimal action, in the guise of the geodesic principle of least length will appear now to describe the internal representation of physical motions, as providing the dynamics of the internalized constraints given by the symmetries. However, rather than such a teleological principle be a determinant of the most fundamental form of motion which corresponds to the variational principle of Classical Mechanics, actually for pointlike systems, in Shepard's setting for the case of the motion of extended objects presenting either symmetries or asymmetries this requires another entirely different formulation which incorporates both cases.

It requires the introduction of a geometric-analytic method to compare the representation from one point of the manifold in which the object is represented to that in another point, to assess the invariance of the whole representation. This requires on the one hand a calculus on manifolds , say of differential forms, tensors, vector fields or Clifford multiforms, and a rule of translation of them from one any given point to the other. In the case in point, each object is represented in a six-dimensional manifold, with three dimensions for the position, and three dimensions to characterize the rotation that the perceived object has suffered. This method is called a Cartan connection, or still an affine connection, whenever the symmetry group is the group of affine transformations (Wikipedia, Affine Connections; ibid, Torsion). This connection has torsion, and not necessarily has an element related to a non-trivial metric which can be the curvatureless Euclidean metric (Goenner,2004); it is intimately related to selfreference and heteroreference (Rapoport, 2013, 2016 I). In General Relativity such a connection is fully dependent on the non-trivial metric on spacetime, and has by default a null torsion; it is called the Levi-Civita connection. In the latter case particularly important motions are the geodesics, the shortest paths between any two points, which for the affine connections are the straighttest curves, the so-called autoparallels.

This brings the rotations into the modeling, actually angular momentum and in the case of pointlike objects it is spin, which quantizes the underlying geometrical manifold. Ultimately, it brings to the modelization the non-orientability of either the manifold where the motions are modeled, or of the cognitive map produced by these motions (Carlton & Shepard,2000). Indeed, geometrical torsion becomes a generator of non-orientability ; see figs. 8 and 10 (Rapoport,2013). Remarkably, the mathematical formalism where the symmetries are given by semidirect sums of Lie symmetry groups of affine transformations (rotations and translations) and the associated affine connections with torsion is the same one for the cognitive modeling from the motions of extended objects, either asymmetric or symmetric, with that of Classical Mechanics for pointlike particles moving in the physical spacetime with torsion (Rapoport &

Sternberg,1984): This surmounts the minimal length principle of variational principles in Classical Mechanics which becomes a particular case of null-spin particles free motion.

Following Velmans' Reflexive Monism we can express it as the cognitive modelization of the perception that arises from the motion of extended objected, either symmetric or asymmetric, as experienced by the seeing subject, is homological to the modelization of motion of space-time particle with spin, as objectively independent-of-the-observer yet in a torsion field background, and viceversa. The physical motion on spacetime as if objective, of particles with symmetries, is introjected as the psychological motion of extended rotating objects, with the torsion now being introjected as the psychological motion of extended moving objects producing the non-orientability of the modeling. This clearly evokes Shepard's Complementarity Principle. Remarkably the early joint attempts by Einstein and Cartan failed due to their lack of understanding of what torsion embodied or was related to at all.

10. THE ERLANGEN PROGRAM, PERCEPTION AND COGNITION, NON-ORIENTABILITY AS MEMORY

Following the mathematization of perception and cognition after Rogerd Shepard, which extends the Erlangen Program due to Felix Klein, based on invariants associated to Lie symmetry groups, crucial to both the foundations of geometry and its implementation as the core of theoretical physics and which we shall retrieve upon examining the modelization process itself, it has been claimed that this program does not account for memory: it is memoryless. In this conception the usual symmetries are to be extended by introducing memory as associated to asymmetry. The latter is not to be conceived as a symmetry-breaking but rather an action which embodies making an additional distinction on a symmetry, whose fundamental form is that of a twist, a torsion which prepares for non-orientability (Leyton, 1992).

Remarkably the latter was unmentioned by Leyton, quite strangely at that since this author claimed that the Erlangen Program was incomplete due to the lack of an embodied memory as its symmetries. So, the selfpenetration of the Klein Bottle, as seen in Figure 3 C is, following Leyton inception of memory as asymmetry, the signature of memory as a selfreferential action, or still, the twist as Moebius strip. Further, we conceive this action and its topology as a primal form of pyschism, actually panpsychism at a most fundamental level of the vacuum prior to the primal distinction, would we conceive it as a Planck-constant--scale spacetime geometry of the vacuum as a quantum foam, teeming with virtual singularities as "black-holes", since the event horizons, the physical frontier of the elicitable physical domain supported by the vacuum, turns out to be a Moebius strip ('t Hooft,2018). More of this below.

Shepard's complementarity operates in two senses. To start with, a relation of complementarity between the object and its representation by the body-mind. "And there is, second, the perhaps deeper, evolutionarily shaped complementary relation that the principles of organization of the internal system (including its rules of formation and of transformation) bear to the corresponding invariances characteristic of our three-dimensional world (including,especially, its invariances of optical projection and of rigid motion)" (p.332). Thus, here the Erlangen Program steps in since for Felix Klein shapes and forms are defined by the Lie groups of symmetries as their invariants. Actually, Sophus Lie conceived his groups of transformations in terms of perception. While Shepard applies this to perception, he further suggests that the same principles apply to language and its structural constraints, and remarkably to music as cognized (Shepard, 1964) which again, elicits a Moebius strip, associated to the organization of perception structured in terms of the Tritone Paradox (Rapoport,2013).

However, as noticed by Shepard: “The internal transformations, like the representations transformed, need have no concrete resemblance to their external counterparts…to speak of “mental rotation’’ is not to imply that there is anything that literally rotates within the subject’s physical brain… The only thing that we claim actually rotates is something that is rather abstract or hypothetical and that is defined in relation to the external world—namely, the orientation in which the external object would be most rapidly discriminated if it were to be physically presented at that moment. Just as I have been proposing that the relation between an internal representation and its external object is one of complementarity, not resemblance, I am now suggesting that the mental operations that transform one representtation into another are complementary, not strictly similar, to the corresponding operations on the external objects”. To which he adds that what is left to embody the modeling of the exterior world is that the transformation of the exterior world motion into an internal image representation must not modify the topology of the model. Shepard frames it as a principle of stability, whereas we must represent an object’s motion in three dimensional space as two-dimensional in its image, understood as “a small rigid transformation in the external world leaves the projected image topologically invariant... Once a particular transformation has been selected… the transformation is carried out in accordance with rules that embody constraints governing corresponding transformations on rigid physical objects”. In other words, the modelization interiorizes the exterior-binded constraints. Furthermore, this is kept all along the motion of the physical object along a trajectory, and the parameter that is to be subsumed in the internal representation is the orientation of the object. The image of the physical path to represent its orientation is the straightest path on the manifold which represents the orientations. The constraints are subsumed in keeping the structural rigidity, so no topological transformations of the object are allowed, in the following sense:Thus, if any topology is to manifest in the image is proper to the modelization by which perception is organized.

However, though he attributed the “internal” symmetries of the modelization to a virtual transformation, both the symmetries of the physical and the cognitive domains are affine transformations of the semidirect kind, since rotations act on translations. This is also the case of the Cartan connections with torsion (Rapoport & Sternberg,1984; Carlton & Shepard,2000).

Yet, while he attributed to the internal representations of rotations a merely “mental” quality, this turned out to be mistaken. First of all, the Klein Bottle with its $4\boldsymbol{\pi}$ symmetry is the metaform of both pattern cognition and recognition without which patterns be that of the physical world or the mental one are nor created nor recognized, particularly supporting brain patterns given by the Laplacian operator harmonics fundamental to patterns of the brain connectome (Atasoy et al,2018). Secondly, the Klein Bottle is the builtin invariance of Gabor wavelets of visual receptive fields supporting holography (Tanaka,1995,1997), so that Reflexive Monism at the neuronal cell level is already embodied. Thirdly, the Klein Bottle topology of the primary visual and somatosensory topographic mappings of the body periphery on the corresponding cortical brain areas (Rapoport 2013). And fourthly, specific to the cognitive modeling of the motions of worldy objects particularly asymmetrical ones where rotations operate, “mental” rotations appear to have an homologous existence in the neuronal dynamics. Indeed, this is clear from the modeling of motor activity as non-linear dynamical systems of coupled oscillators, where behaviour is represented as the strange attractors of this dynamics. This appears to be the case of polyryhthmic motor performance (Jagacinski et al, 2000;Turvey, 1992) and the rotational dynamics of population of motor cells first discovered by Churchland (Lebedev et al,2019) where non-orientability shows to be the case. The rotational motions which Shepard –and later Risset- showed to be the case of the psyche’s making up of images, particularly music (Shepard, 1964; Vernooij et al,2016) as in the Tritone

Paradox with its Moebius strip which is experienced alike Escher's never ending *Ascending and Descending,* appear indeed to have a neurological basis.

The cognitive map produces a well-defined trajectory, each point of which is complementary to a corresponding intermediate stage of transformation of the external object. It is in this sense that the internal process is said to be a complementary analog of the corresponding external process. This trajectory was initially assumed to be a geodesic path, i.e. a shortest path on a curved manifold relative to a metric satisfying certain constraints, given by corresponding the maintenance of the structural rigidity of the external object, and later as an autoparallel curve of a Cartan affine connection with torsion (Carlton & Shepard,1990). Whatever the case is, the path supports the motion and the transformation. For this Shepard first adopted a teleological principle of least action from Classical Mechanics which has been extended to theoretical physics at large in terms of the so-called variational problems. This is fully coherent with the current approach in terms of free energy already mentioned repeatedly, but stays short in relation with the introjection of the general setting for the symmetries and their introjection through complementarity which does not require it. The Cartan connections and the general geometrical modelization of vision is reivindicated in the developments that follow the consideration that the brain is a 'geometrical visual engine' (Petitot,2003).

11. THE MENTAL MODELIZATION CONSTRUAL, PROJECTIVITY AND NONORIENTABILITY

Given the supradual ontology that actually construes our cognition and the physical world as a unity, we want to draw our attention to the construal of the models that support this, and to study the geometrical and topological principles that support this construal.

Shepard departed from empirical findings to model the body-mind ("internal") processes by which we represent the worldly "exterior" objects, events and processes (OAHs), and the relations between the latter. Notably, music and its cognition played a crucial role to the development of this theory. The term "complementarity" introduced by Shepard, recalling the wave-particle complementarity of the Bohr conception of the quantum world, arose first to distinguish it from a relation of physical resemblance. In doing so, he argued that symmetries of space which code for the invariant properties of symmetry transformations, particularly of motion and perception, play a crucial role, in that they are "complementary" to the invariances by the "internal" symmetry transformations of the representations. In doing this he departed from what theoretical physics has come to conceive its very conceptual mathematical basis for its formulation, the groups of continuous Lie symmetries, called after the name of Sophus Lie. Together with the principles of minimization of Maupertius and Euler, which became the 19th century instruments for the formulation of classical mechanics and the 20th century further used as a principle of symmetry for theoretical physics at large.

Remarkably, as observed by Shepard in his theory of complementarity, the most important constraints are those that arose from the perceived locally Euclidean three-dimensionality of space and of the objects placed as if contained in it, and that these appear through two-dimensional surfaces of their boundary, both of them as characteristics of the perceived environment and our actions on them. Additionally these objects tend to conserve their semirigid shape under said transformations. The final constraint is given by the group of transformations of the locally Euclidean three dimensional space, the Euclidean group, its symmetry group.To elicit the topology that supports the body-mind for spatial visual perception Shepard finally choose the special Euclidean group of rigid motions, the Euclidean motions, of rotations and translations, Since the rotations act on the translations, then E(3) is the affine group expressed as a semidirect sum we already mentioned,the symmetry goup of the kinematic geometry of Shepard. E(3) is the final constraint considered by Shepard, and also appears in the Projective

Consciousness Model of Willford-Rudrauf- to be introduced below. What will come out of this is the blowup of the origin, the Moebius strip, already discussed as the primal morphogenesis arising from a singularity, the origin. The creational-recreational act of the world as the Renaissance Masters (Leonardo, Alberti, Durer) putting up projective geometry.

Particularly important is the property of invariance under rotational transformations of say an object in space, which is represented by the invariance under "mental" rotations of its representation. Remarkably, in this case, the said complementarity by Shepard is a resemblance, indeed.What is most surprising is that the physical rotations of the objects and their projective geometry representation produce an identification of antipodal points of the two-dimensional sphere of the angular parameters internal representation (Shepard, 1982;Willford et al,2012; Rudrauf et al,2017) which yields a nonorientable Moebius strip as the resultant of this invariance (Rapoport, 2013), and was also observed in the representation of irregular planar polygons by Shepard. This nonorientability actually denotes the point at infinity of projective geometry, as seen by an observer as the horizon of his perceived scape (Rapoport, 2013) and which in the Projective Consciousness Model of Willford et al, stands for the Self, just as shown by Rapoport. However there is a difference between these theories with regards to dimension of the space of consciousness. In the case of PCM, the ambient space from which the ambient space is Euclidean 3d space and its associated projective space is two-dimensional, actually the 2-dimensional sphere, in contrast with the 3-dimensional sphere of PCM, apparently from mimicking the four-dimensionality of physical spacetime. We shall return to this issue of the multidimensionality of the bodybrain. Shepard argues that "the internal representation is not of the inherent three-dimensional structure of the object itself but only of that structure as seen from a particular direction" and thus the internal representation is 2-dimensional by this reduction by a particular direction, rather than 3 in PCM. Thus, there is no resemblance with the actual physical object but rather the representation produces a "mesh", a complementarity which according to Shepard is of topological nature, rather than a geometrical one.

Therefore, the internal representation to Shepard is two-dimensional and achieved through projection, and still, has an inverse which is the retrojection of the projection, which is mandatory to survival through the action-perception unity, which Turvey related to a Moebius strip topology of their complementarity. Now, the physical object is not static but moving, and thus the projection of its motion produces a trajectory on the representation two.dimensional plane along which the complementarity of the object and its representation is maintained. Thus, when the object is represented along this trajectory, the antipodal points are identified and thus a Moebius strip is produced moving along the points of the trajectory which Shepard took initially to be a minimal length path, a geodesic path, later an autoparallel.

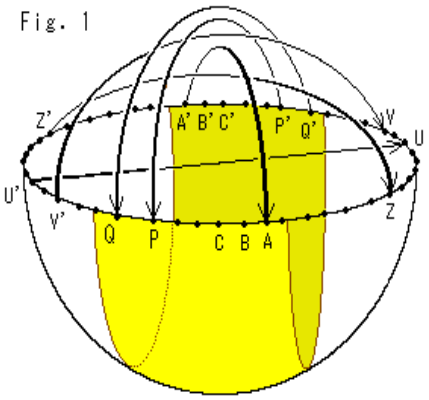

Figure 1.Two points at opposite ends of a sphere are called **antipodal points**. On the edge of a hemisphere there are countless anti-podal points. **All antipodal points on the edge of the hemisphere made by central projection from the ground must be identified**. That is, the ground that is spreaded infinitely has something like antipodal points. Such plane is **projective plane.** Though a visual model of the entire projective plane can not be made, we can get a part of it by cutting the hemisphere like the yellow belt. It is a well-known **Möbius strip**

Figure 6. (Reproduced from (Rapoport, 2013, fig.1)), CC

Then the object through object conservation and minimal energy requirement is conceived as a single point moving along this trajectory, or as we shall see, along a Moebius strip. Remarkably, it is known that a neural bounded network in the hippocampus of the rat supports a head orientation map, which given the finiteness of extensión of the network requires the imposition of periodic boundary conditions which, curiously, were assumed to be that of a 2-torus instead of the Moebius (Mc Naughton et al,2006).

Indeed, everything that is perceived ultimately is represented as a two-dimensional surface and is perceived as a faceted partiality, a profile as seen from one particular angle (Shepard, 1981; Willford et al,2012). On the other hand, consciousness itself is always facetless and a fluid whole-ness, which sustains all possible perceptual facets and the perspectives that elicit the particular percepts, and still the experience and intuition of the unity. As such, consciousness, as locational, as topology of Being which supports all particularities, is alike the integrality of a Moebius strip which by 180ª twisting has one single line containing all possible locations and perspectives of particular perceptions, or the Klein bottle by two twists gluing that single side to itself keeping the same orientation.Thus, under projective antipodal identifications of points of the horizon, the torsion is produced, though this identification might be virtual, as consciousness is and still the perspectiveness as the possibility of enacting these perspectives are united in the identification of the infinite points of the horizon as a unified profileless phenomena.

Thus, consciousness alike time contains all possible locations and objects at one single point very much alike Heidegger's timespace- that arises from the nonorientability, actually the 5d standing wave representation of the Klein Bottle, which supports the process of pattern formation and recognition in the first place, which when considered in the 3d locally Euclidean space of experience, is nothing but a projection to a 2d pattern formation and recognition where only 2d sections of the solid body can be seen and recognized as partial sections of a whole-ness whose perception cannot be reified. "In fact the Klein Bottle representation in $\mathbb{R}$" is the projection onto the spherical harmonics of order 1 and 2 of the essentially 1D (i.e. isotropic) signal f(x) = cos(ρ <**u**,**x**>)This is quite remarkable: The form of forms is nothing else than the projection to order 1 and 2 in the space of spherical harmonics of the sinusoidal 1D signal with vector orientation that arises from the fringes of intrinsically 1D images. Of course, this is not exclusive to images, but to any such sinusoidal signal, and specifically, that arises from the fact that the orientation can not be uniquely defined on the boundary of the domains under examination. Indeed, the point is that fringe signals have a definite orientation which is

rendered by a 2D phase. An image of dimension n for its representation as an essentially 1D signal, i.e. a signal whose representation requires only one dimension due to the isotropy of the phase, is such that *antipodal phases are identified as having the same signal*; we have repeatedly presented this phenomenon in music cognition, geophysical configurations and morphogenesis, etc (Rapoport 2013). So this is about about multivaluedness and superposition that boundaries/distinctions produce, topological entanglement –or still semiotic entanglement (Merrell) which is ontologically prior vis-a-vis quantum entanglement, which is precisely the essence of the Klein Bottle Logic and its Matrix Logic representation (Stern, Rapoport 2011). Let us now present this four-dimensional, as presented as a dynamical figure in four Euclidean coordinates (x,y,z,t):
https://en.wikipedia.org/wiki/Klein_bottle#/media/File:Klein_bottle_time_evolution_in_xyzt-space.gif

Note that it has a wholeness that stems from the selfpenetration cross-section circle and that it blows up to return to it through constructing the selfpenetration cylinder to later disappearing altogether as if inexistent and back. Thus,it represents the selfgenerative Klein Bottle as emanating from the selfpenetrating cross-section which appears to organize the wholeness through its particular standing and to selferase as if in its creative organizing agency it disappears. Wholeness as emanating from holeness to return to it, erasing both to renaissance both.

These characterizations of consciousness as associated to nonorientability are absent in Willford et al, who choose the origin of 4d Euclidean space, for its immovility, to be the representing locus for consciousness as
"Consciousness in its self-awareness is more like the immovable origin of a projected space (which, as we shall see, in a sense to be made more precise and definite below, coincides with the (hyper)horizon of whole field or space). While the coordinates of the objects-as-profiled appearing in the space can change—the objects can undergo magnification and rotation, and the entire set of perceived objects in the space can undergo transformations of various kinds—the origin itself cannot. The origin is, as we shall see, itself a sort of virtual point. (Of course, to the extent that consciousness is a real process going on in brains localized in real space it and its origin point do move around in that real space. One simply must be careful here to distinguish the virtual space we project from the real physical space in which we reside. In the natural attitude of direct realism, we spontaneously identify the projected space with the real space, but we believe this is merely a generally adaptive illusion…The origin of this space of phenomenal representation cannot become a set of points in the projected space *other than* the origin". However, we repeat rather than the origin for its omission what is at stake is topological entanglement which manifests as semiotic entanglement, and thus the wholeness of systems operate by and we and the world co-constitute ourselves and colaborate through our being as interpretant signs (Merrell).

However, as the authors concede, the origin of consciousness cannot be objectified as we do with the objects contained in space, "we may say that we inherently "take a perspective" on our own consciousness all the time. But that is to be interpreted as meaning that consciousness is self-given all the time. While it is true that this self-givenness contains within it the seeds of all other forms of self-distantiation, the point in this section has been to articulate the way in which it is *different from* the givenness of perceived physical objects". Actually, in that objectification by projectivity in which the eye of the subject is indeed placed as the origin, this point is omitted from the projective space, so we shall identify the subject as the antipodal points emanating from then origin being identified. It is this imaginal identification what makes

space objective, not the origin itself. On the other hand, as discussed in ¨(Rapoport 2013), the line at infinity define by the abstract definition of projective space ceases to be the actual line at infinity defined by projection having the eye for origin, whenever the object is such that its boundary cannot be located locally in a flat space as a tangent space approximation of the actual spherical surface of Earth. They turn to be produced by the embedding of a Moebius strip in projective space.

We turn to introduce the gnomonic projection to introduce the issue of infinity in projective geometry and the Möbius strip.

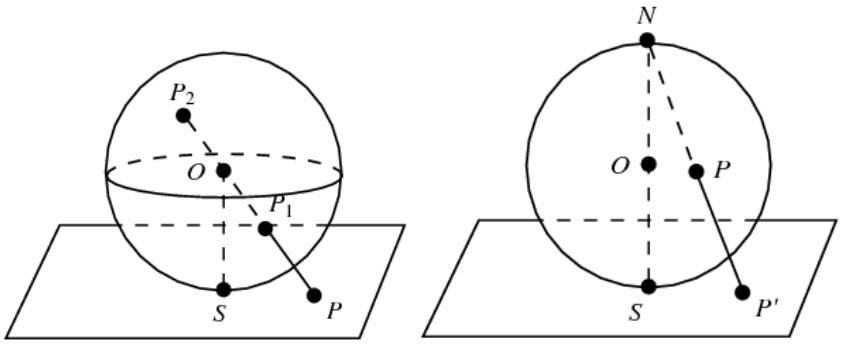

Figure 7: The gnomonic and stereographic projection of projective geometry and complex analysis, respectively.

We introduced the gnomonic projection in figure 7. The gnomonic projection is a nonconformal map projection obtained by projecting point P_1or P_2 on the surface of sphere from a sphere's center O to point P in a plane that is tangent to a point S. In the above figures, S is the south pole, but for the gnomonic projection can be any point on the sphere. Since this projection obviously sends antipodal points P_1or $P_2$to the same point P in the plane, it can only be used to project one hemisphere at a time. This is the why in Figures 6 we have also used the lower hemisphere. In a gnomonic projection, great circles are mapped to straight lines as shown in figure 6. The gnomonic projection represents the image formed by a spherical lens, say the eye,and is sometimes known as the rectilinear projection. It is to be distinguished from the (conformal) stereographic projection of the Riemann sphere, in which the point of projection is the North Pole replacing the center 0, and the contact point with the surface can no longer be arbitrary but the South Pole, identified in the complex plane S as the origin of the complex numbers. The naming of the Gnomonic projection comes from the Greek gnomon, the pedestal of a sundial (solar clock), which indeed it is not placed on the centre of Earth but on any arbitrary point of its surface, and by which the shade of the Sun projecting on a surface would be marked to identify the time of the day. It was this projection which was used not only to keep track of the time of the day, but the tracking of the seasons due to the precession of the equinoxes derived from the wobbling of Earth's inclined axis further moving along the circumsolar elliptic orbit. The overall graph of this annual projection is the Analemma, the Möbius strip projected on the surface of Earth (Rapoport, 2013).

Consider now the following image which represents the view from a camera eye of a triangularfigure projected on a two-dimensional screen, as occurs on the V1 area of the neurocortex in which the visual stimuli are projected. As stressed repeatedly by Shepard upon introducing his account of complementarity of perception, it is the projective transformation of an actual 3d stimuli onto a two dimensional plane the main constraint in terms of which an ecological theory of perception is to be considered, and the inverse retrojection what completes

the representation of this stimuli as a regression required to complete the action-perception cycle, which actually Turvey proposed to be a Moebius strip (Turvey,2004).

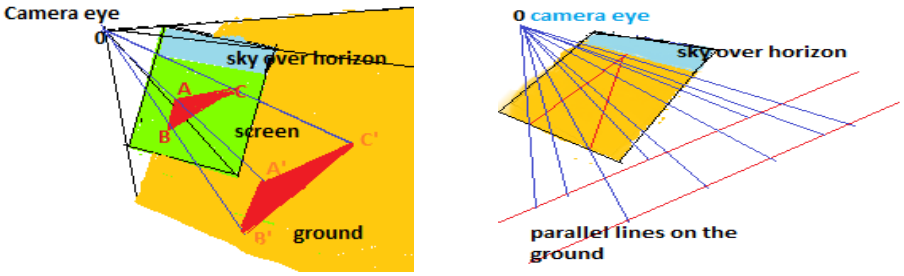

Figure 8: The gnomonic projection and the line at infinity. From (Rapoport,2013) CC

So we imagine the projection of a point **0**, which now places the role of the centre of the sphere which we remove and still we take for the surface on which we practice the projection, to be a section of the tangent plane to Earth's surface.. The camera is the projection center and takes the picture of triangle **A'B'C'** on the ground. This ground is supposed to be the tangent plane. The light green area is the view from the camera. Black rays from the camera indicate stereo area (pyramid) of the viewfinder. The square board fitted to black rays is to show what we can look at through the camera. Let us call the square board the -screen. The photo of triangle **A'B'C'** on the ground is taken as triangle **ABC** on the-screen. Blue rays are projective lines of vertices of triangles. Consider now a pair of parallel lines on the surface and we lift them as if derived from the gnomonic projection of two lines drawn in red. These lines do intersect, in fact they do so on the horizon, on which we place the vertex/intersection point, with the horizon drawn in light green and the sky in light blue. Thus, the horizon which in projective geometry is called the line at infinity is not such line in the actual world in which the tangent plane, flat as it is ceases to provide a good approximation to the curved surface of the sphere. It is neither an imaginary line, but in the definition of mathematicians, as we already saw in identifying the line at infinity in the gnomonic projection as the equator of the sphere. Instead of the elusive imaginary line at infinity, we can identify it in the panel screen as the boundary between Earth and sky. It is no line at infinity at all. So again, what physicists would name a divergence, has a very concrete manifestation, and the very notion of infiniteness is called into question. Of course, we can take the camera's eye for the North Pole instead of its substitution for the center of the sphere, and the conclusions still apply to the stereographic projection; thus the conclusions do not require we have a conformal (stereographic) nor a non-conformal (gnomonic) projection. In fact, for visual perception, in which the centre of the retina plays the role of the point of organization of incoming light, it is the conformal mapping that applies as we discussed in (Rapoport, 2013). So, whether it is a camera eye or a human eye, whenever the twodimensional image of the cannot be localized in a surface which can be properly approximated by a plane, then the line at infinity of projective space is not the line at infinity given by the horizon defined by the boundary of Earth and the sky, very much finite at that. The line of infinity produced by the 3d projection on a plane, that is the central constraint in ecological perception, can be thought as the manifestation of the non-local embedding of the Möbius strip on the projective plane, as discussed in (Rapoport, 2013). We want to return to the argument of the Projective Consciousness theory of Willford et al, that the origin of projective space stands for the source of consciousness. While the origin is transformed by the inversion mapping of the conformal transformation into infinite, actually the North Pole in the

conformal mapping representation of above, again, this would be the case whenever the figure being seen by the eye, be that of a camera or human eye, would not stand for a realization of the objectivity of projection,which instead is the case as the embedding of the Moebius strip in locally Euclidean space, as actually empirically verified by Shepard in the case of irregular polygons. For a thorough examination and illuminating vistas of nonorientability, hyperbolic geometry, asymptotics and the infinity line see (Rapoport,2013).

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